

INTEGRALS

INTRODUCCIÓ

Llibre d'exercicis adreçat als alumnes de Batxillerat Tecnològic per millorar el seu nivell de coneixements i procediments en l'àrea de Matemàtiques

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ÍNDEX

1. INTEGRAL INDEFINIDA - CALCUL DE PRIMITIVES

<i>1.1</i>	<i>Immediates</i>	<i>4</i>
<i>1.2</i>	<i>Quasi_Immediates</i>	<i>5</i>
<i>1.3</i>	<i>Per Parts</i>	<i>16</i>
<i>1.4</i>	<i>Racionals</i>	<i>22</i>
<i>1.5</i>	<i>Altres</i>	<i>26</i>

2. INTEGRAL DEFINIDA

<i>2.1</i>	<i>Àrea d'un recinte</i>	<i>34</i>
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INTEGRAL INDEFINIDA - CALCUL DE PRIMITIVES

1. IMMEDIATES 2. QUASI IMMEDIATES 3. INTEGRACIÓ PER PARTS 4. RACIONALS 5. ALTRES

<div style="border: 1px solid gray; background-color: #ccc; padding: 5px; display: inline-block; margin-bottom: 5px;">1</div> <p>IMMEDIATES</p>	
<p>1.1</p> $\int x^n dx = \frac{x^{n+1}}{n+1} + c$	<p>1.2</p> $\int f(x)^n f'(x) dx = \frac{f(x)^{n+1}}{n+1} + c$
<p>1.3</p> $\int \frac{1}{x} dx = L x + c$	<p>1.4</p> $\int \frac{1}{f(x)} f'(x) dx = L f(x) + c$
<p>1.5</p> $\int e^x dx = e^x + c$	<p>1.6</p> $\int e^{f(x)} f'(x) dx = e^{f(x)} + c$
<p>1.7</p> $\int a^x dx = \frac{a^x}{L a} + c$	<p>1.8</p> $\int a^{f(x)} f'(x) dx = \frac{a^{f(x)}}{L a} + c$
<p>1.9</p> $\int \sin x dx = -\cos x + c$	<p>1.10</p> $\int \sin f(x) f'(x) dx = -\cos f(x) + c$
<p>1.11</p> $\int \cos x dx = \sin x + c$	<p>1.12</p> $\int \cos f(x) f'(x) dx = \sin f(x) + c$
<p>1.13</p> $\int \frac{1}{\cos^2 x} dx = \operatorname{tg} x + c$	<p>1.14</p> $\int \frac{1}{\cos^2 f(x)} f'(x) dx = \operatorname{tg} f(x) + c$
<p>1.15</p> $\int \frac{1}{\sin^2 x} dx = -\operatorname{cotg} x + c$	<p>1.16</p> $\int \frac{1}{\sin^2 f(x)} f'(x) dx = -\operatorname{cotg} f(x) + c$
<p>1.17</p>	<p>1.18</p>

$\int \frac{1}{1+x^2} dx = \text{arc tg } x + c$	$\int \frac{1}{1+f(x)^2} f'(x) dx = \text{arc tg } f(x) + c$
1.19 $\int \frac{1}{\sqrt{1-x^2}} dx = \text{arc sin } x + c$	1.20 $\int \frac{1}{\sqrt{1-f(x)^2}} f'(x) dx = \text{arc sin } f(x) + c$

2

QUASI IMMEDIATES

2.1. $\int x^3 dx$	Sol. $\frac{x^4}{4} + c$
2.2. $\int \frac{x^3}{3} dx$	Sol. $\frac{x^4}{12} + c$
2.3. $\int \frac{x^4}{6} dx$	Sol. $\frac{x^5}{30} + c$
2.4. $\int (x^3 + 3) dx$	Sol. $\frac{x^4}{4} + 3x + c$
2.5. $\int (x^2 + 2x - \frac{1}{x}) dx$	Sol. $\frac{x^3}{3} + x^2 - \ln x + c$
RAONAMENT	
$\int (x^2 + 2x - \frac{1}{x}) dx = \int x^2 dx + 2 \int x dx - \int \frac{1}{x} dx = \frac{x^3}{3} + x^2 - \ln x + c$	
2.6.	Sol.

$\int \frac{x^3 - x^2 + 1}{x} dx$	$\frac{x^3}{3} + \frac{x^2}{2} + \ln x + c$
2.7. $\int \frac{dx}{x^2}$	<i>Sol.</i> $-\frac{1}{x} + c$
2.8. $\int \frac{dx}{x^5}$	<i>Sol.</i> $-\frac{1}{4x^4} + c$
2.9. $\int \frac{x^4 - 2x + 3}{x^6} dx$	<i>Sol.</i> $-\frac{1}{5x^5} + \frac{1}{2x^4} - \frac{3}{5x^5} + c$
2.10. $\int \frac{4\sqrt[3]{x}}{3} dx$	<i>Sol.</i> $\sqrt[3]{x^4} + c$
RAONAMENT	
$\int \frac{4\sqrt[3]{x}}{3} dx = \frac{4}{3} \int x^{1/3} dx = \frac{4}{3} \frac{x^{4/3}}{4/3} = x^{4/3} + c = \sqrt[3]{x^4} + c$	
2.11. $\int \frac{dx}{\sqrt[4]{x}}$	<i>Sol.</i> $\frac{4\sqrt[4]{x^3}}{3} + c$
2.12. $\int \left(\frac{8}{3} \sqrt[3]{x} + 3\sqrt{x} \right) dx$	<i>Sol.</i> $2\sqrt[3]{x^4} + 2\sqrt{x^3} + c$
2.13. $\int \sqrt[3]{x} (\sqrt{x} + 1) dx$	<i>Sol.</i> $\frac{6}{11} \sqrt[6]{x^{11}} + \frac{3}{4} \sqrt[3]{x^4} + c$
2.14. $\int (x^2 - 2 \sin x + 8 \cos x) dx$	<i>Sol.</i> $\frac{x^3}{3} + 2 \cos x + 8 \sin x + c$
2.15.	<i>Sol.</i>

$\int \left(e^x + \frac{1}{x} \right) dx$	$e^x + \ln x + c$
<i>RAONAMENT</i>	
$\int \left(e^x + \frac{1}{x} \right) dx = \int e^x dx + \int \frac{1}{x} dx = e^x + \ln x + c$	
2.16. $\int \frac{dx}{x \sqrt{x}}$	<i>Sol.</i> $\frac{-2}{\sqrt{x}} + c$
2.17. $\int \frac{(x+1)(x^2+3)}{x^3} dx$	<i>Sol.</i> $x + \ln x - \frac{3}{x} - \frac{3}{2x^2} + c$
2.18. $\int (\sec^2 x + \cos x + x) dx$	<i>Sol.</i> $\operatorname{tg} x + \sin x + \frac{x^2}{2} + c$
2.19. $\int \operatorname{tg}^2 x dx$	<i>Sol.</i> $\operatorname{tg} x - x + c$
2.20. $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx$	<i>Sol.</i> $\frac{2\sqrt{x^3}}{3} + 2\sqrt{x} + c$
<i>RAONAMENT</i>	
$\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx = \int x^{1/2} dx + \int x^{-1/2} dx = \frac{x^{3/2}}{\frac{3}{2}} + \frac{x^{1/2}}{\frac{1}{2}} + c =$	
$\frac{2\sqrt{x^3}}{3} + 2\sqrt{x} + c$	
2.21. $\int \frac{\cos^2 x - \sin^2 x}{\sin^2 x \cos^2 x} dx$	<i>Sol.</i> $-\operatorname{cotg} x - \operatorname{tg} x + c$
2.22.	<i>Sol.</i> $e^x + \ln x + c$

$\int e^x \left(1 + \frac{e^{-x}}{x} \right) dx$	
2.23. $\int 5^x 3^x dx$	<i>Sol.</i> $\frac{15^x}{\ln 15} + c$
2.24. $\int \left(\frac{1}{\sqrt{1-x^2}} - \frac{3}{1+x^2} \right) dx$	<i>Sol.</i> $\arcsin x - 3 \operatorname{arctg} x + c$
2.25. $\int \frac{dx}{\sin^2 x \cos^2 x}$	<i>Sol.</i> $\operatorname{tg} x - \operatorname{cotg} x + c$
<i>RAONAMENT</i>	
$\int \frac{dx}{\sin^2 x \cos^2 x} = \int \frac{1}{\sin^2 x} dx + \int \frac{1}{\cos^2 x} dx = -\operatorname{cot} gx + \operatorname{tg} x + c$	
2.26. $\int \frac{2 - \sin^3 x}{\sin^2 x} dx$	<i>Sol.</i> $-2 \operatorname{cotg} x + \cos x + c$
2.27. $\int \frac{dx}{3x + 2}$	<i>Sol.</i> $\frac{1}{3} \ln 3x + 2 + c$
2.28. $\int \frac{dx}{3 - x}$	<i>Sol.</i> $-\ln 3 - x + c$
2.29. $\int \frac{x dx}{2 + x^2}$	<i>Sol.</i> $\frac{1}{2} \ln 2 + x^2 + c$
2.30. $\int \frac{2 dx}{(x + 1)^3}$	<i>Sol.</i> $\frac{-1}{(x + 1)^2} + c$
<i>RAONAMENT</i>	

$\int \frac{2 dx}{(x+1)^3} = 2 \int t^{-3} dt = 2 \frac{t^{-2}}{-2} + c = -\frac{1}{t^2} + c = \frac{-1}{(x+1)^2} + c$ $t = x+1 \quad dt = dx$	
2.31.	<i>Sol.</i>
$\int \frac{x^2 dx}{1+x^3}$	$\frac{1}{3} \ln 1+x^3 + c$
2.32.	<i>Sol.</i>
$\int \frac{\sin 2x dx}{3 + \sin^2 x}$	$\ln 3 + \sin^2 x + c$
2.33.	<i>Sol.</i>
$\int \frac{(x-3) dx}{\sqrt{x^2 - 6x + 1}}$	$\sqrt{x^2 - 6x + 1} + c$
2.34.	<i>Sol.</i>
$\int e^x \sqrt{2 + e^x} dx$	$\frac{2 \sqrt{(2 + e^x)^3}}{3} + c$
2.35.	<i>Sol.</i>
$\int \frac{\ln x}{x} dx$	$\frac{\ln^2 x}{2} + c$
RAONAMENT	
$\int \frac{\ln x}{x} dx = \int t dt = \frac{t^2}{2} + c = \frac{\ln^2 x}{2} + c$ $t = \ln x \quad dt = \frac{1}{x} dx$	
2.36.	<i>Sol.</i>
$\int x^2 \sqrt{x^3 + 1} dx$	$\frac{2 \sqrt{(x^3 + 1)^3}}{9} + c$
2.37.	<i>Sol.</i>
$\int \sin 5x dx$	$-\frac{1}{5} \cos 5x + c$
2.38.	<i>Sol.</i>

$\int 6x \cos x^2 dx$	$3 \sin x^2 + c$
2.39. $\int \frac{\cos x dx}{1 + \sin^2 x}$	<i>Sol.</i> $\arctg (\sin x) + c$
2.40. $\int \frac{dx}{\cos^2 x \sqrt{1 - \operatorname{tg}^2 x}}$	<i>Sol.</i> $\operatorname{arc} \sin (\operatorname{tg} x) + c$
RAONAMENT	
$\int \frac{dx}{\cos^2 x \sqrt{1 - \operatorname{tg}^2 x}} = \int \frac{dt}{\sqrt{1 - t^2}} = \operatorname{arcsin} t + c = \operatorname{arc} \sin (\operatorname{tg} x) + c$ $t = \operatorname{tg} x \quad dt = \frac{1}{\cos^2 x} dx$	
2.41. $\int x^4 e^{x^5} dx$	<i>Sol.</i> $\frac{e^{x^5}}{5} + c$
2.42. $\int \frac{(4 x^3) dx}{1 + x^8}$	<i>Sol.</i> $\operatorname{arc} \operatorname{tg} x^4 + c$
2.43. $\int 2^x dx$	<i>Sol.</i> $\frac{2^x}{\ln 2} + c$
2.44. $\int \frac{dx}{x^2 + 9}$	<i>Sol.</i> $\frac{1}{3} \operatorname{arc} \operatorname{tg} \frac{x}{3} + c$
2.45. $\int e^{7x} dx$	<i>Sol.</i> $\frac{e^{7x}}{7} + c$
RAONAMENT	
$\int e^{7x} dx = \frac{1}{7} \int e^t dt = \frac{e^t}{7} + c = \frac{e^{7x}}{7} + c$	

$t = 7x \quad dt = 7dx$	
2.46. $\int (e^x + e^{-x}) dx$	<i>Sol.</i> $e^x - e^{-x} + c$
2.47. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}$	<i>Sol.</i> $2e^{\sqrt{x}} + c$
2.48. $\int \frac{\cos x}{e^{\sin x}} dx$	<i>Sol.</i> $-e^{-\sin x} + c$
2.49. $\int \frac{dx}{\sqrt{25 - x^2}}$	<i>Sol.</i> $\arcsin \left(\frac{x}{5} \right) + c$
2.50. $\int \frac{dx}{2x^2 + 9}$	<i>Sol.</i> $\frac{1}{3\sqrt{2}} \arctg \left(\frac{\sqrt{2}x}{3} \right) + c$
RAONAMENT	
$\int \frac{dx}{2x^2 + 9} = \frac{1}{9} \int \frac{dx}{\left(\frac{\sqrt{2}}{3}x \right)^2 + 1} = \frac{1}{3\sqrt{2}} \int \frac{\frac{\sqrt{2}}{3}dx}{1 + \left(\frac{\sqrt{2}}{3}x \right)^2} = \frac{1}{3\sqrt{2}} \int \frac{dt}{1+t^2} =$ $\frac{1}{3\sqrt{2}} \arctgt + c = \frac{1}{3\sqrt{2}} \arctg \left(\frac{\sqrt{2}x}{3} \right) + c$ $t = \frac{\sqrt{2}}{3}x \quad dt = \frac{\sqrt{2}}{3}dx$	
2.51. $\int (2x + 5)^9 dx$	<i>Sol.</i> $\frac{(2x + 5)^{10}}{20} + c$
2.52.	<i>Sol.</i>

$\int \frac{(\operatorname{arctg} x)^3 dx}{1+x^2}$	$\frac{(\operatorname{arc} \operatorname{tg} x)^4}{4} + c$
2.53. $\int \sin^5 x \cos x dx$	<i>Sol.</i> $\frac{\sin^6 x}{6} + c$
2.54. $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}} dx$	<i>Sol.</i> $3 \sqrt[3]{\sin x} + c$
2.55. $\int \frac{dx}{x \ln x}$	<i>Sol.</i> $\ln \ln x + c$
RAONAMENT	
$\int \frac{dx}{x \ln x} = \int \frac{dt}{t} \ln t + c = \ln \ln x + c$	
$t = \ln x \quad dt = \frac{dx}{x}$	
2.56. $\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$	<i>Sol.</i> $2 \sin \sqrt{x} + c$
2.57. $\int (e^x + e^{-x})^2 dx$	<i>Sol.</i> $\frac{e^{2x}}{2} + 2x - \frac{e^{-2x}}{2} + c$
2.58. $\int \frac{dx}{(\arccos x)^3 \sqrt{1-x^2}}$	<i>Sol.</i> $\frac{(\operatorname{arc} \cos x)^{-2}}{2} + c$
2.59. $\int \frac{1 + \ln x}{5 + x \ln x} dx$	<i>Sol.</i> $\ln 5 + x \ln x + c$
2.60. $\int \frac{\operatorname{tg}^2 x + \operatorname{tg} x}{\cos^2 x} dx$	<i>Sol.</i> $\frac{\operatorname{tg}^3 x}{3} + \frac{\operatorname{tg}^2 x}{2} + c$

RAONAMENT

$$\int \frac{\operatorname{tg}^2 x + \operatorname{tg} x}{\cos^2 x} dx = \int (t^2 + t) dt = \frac{t^3}{3} + \frac{t^2}{2} + c = \frac{\operatorname{tg}^3 x}{3} + \frac{\operatorname{tg}^2 x}{2} + c$$

$$t = \operatorname{tg} x \quad dt = \frac{dx}{\cos^2 x}$$

2.61.

$$\int \sqrt{\cos x} \sin x dx$$

Sol.

$$\frac{-2 \sqrt{(\cos x)^3}}{3} + c$$

2.62.

$$\int e^{e^x} e^x dx$$

Sol.

$$e^{e^x} + c$$

2.63.

$$\int e^x \cos e^x dx$$

Sol.

$$\sin e^x + c$$

2.64.

$$\int \frac{dx}{(x+1)^2 + 1}$$

Sol.

$$\operatorname{arc} \operatorname{tg} (x+1) + c$$

2.65.

$$\int \frac{\sin^3 x}{\sqrt{\cos x}} dx$$

Sol.

$$-2 \sqrt{\cos x} + \frac{2 \sqrt{(\cos x)^5}}{5} + c$$

RAONAMENT

$$\int \frac{\sin^3 x}{\sqrt{\cos x}} dx = \int 2 \frac{(1 - \cos^2 x) \sin x}{2 \sqrt{\cos x}} dx = -2 \int (1 - t^4) dt = -2t + \frac{2t^5}{5} + c =$$

$$-2 \sqrt{\cos x} + \frac{2 \sqrt{(\cos x)^5}}{5} + c$$

$$t = \sqrt{\cos x} \quad dt = \frac{-\sin x}{2\sqrt{\cos x}}$$

2.66.

$$\int \sin \ln x \frac{dx}{x}$$

Sol.

$$-\cos(\ln x) + c$$

2.67. $\int \frac{x^2 dx}{2 + x^6}$	<i>Sol.</i> $\frac{1}{3\sqrt{2}} \operatorname{arc\,tg} \left(\frac{x^3}{\sqrt{2}} \right) + c$
2.68. $\int \frac{dx}{x \sqrt{1 - \ln^2 x}} dx$	<i>Sol.</i> $\operatorname{arc\,sin} (\ln x) + c$
2.69. $\int \frac{\sqrt{3 + \sqrt{x}}}{\sqrt{x}} dx$	<i>Sol.</i> $\frac{4 \sqrt{(3 + \sqrt{x})^3}}{3} + c$
2.70. $\int \frac{\cos^3 x}{\sin^3 x} dx$	<i>Sol.</i> $-\frac{1}{2 \sin^2 x} - \ln \sin x + c$
RAONAMENT	
$\int \frac{\cos^3 x}{\sin^3 x} dx = \int \frac{(1 - \sin^2 x) \cos x dx}{\sin^3 x} = \int (t^{-3} - t^{-1}) dt = \frac{t^{-2}}{-2} - \ln t + c =$ $-\frac{1}{2 \sin^2 x} - \ln \sin x + c$ $t = \sin x \quad dt = \cos x dx$	
2.71. $\int x^3 e^{-x^4} dx$	<i>Sol.</i> $-\frac{1}{4} e^{-x^4} + c$
2.72. $\int \frac{e^x}{e^{2x} - 2e^x + 1} dx$	<i>Sol.</i> $-\frac{1}{e^x - 1} + c$
2.73. $\int \frac{x dx}{\sqrt{1 - x^4}}$	<i>Sol.</i> $\frac{1}{2} \operatorname{arc\,sin} x^2 + c$
2.74. $\int x \sqrt{1 + x^2} dx$	<i>Sol.</i> $\frac{1}{3} \sqrt{(1 + x^2)^3} + c$

<p>2.75.</p> $\int \ln(\cos x) \operatorname{tg} x \, dx$	<p>Sol.</p> $-\frac{\ln^2(\cos x)}{2} + c$
<p><i>RAONAMENT</i></p> $\int \ln(\cos x) \operatorname{tg} x \, dx = -\int t \, dt = -\frac{t^2}{2} + c = -\frac{\ln^2(\cos x)}{2} + c$ $t = \ln(\cos x) \quad dt = -\operatorname{tg} x$	
<p>2.76.</p> $\int \frac{\ln(\ln x)}{x \ln x} \, dx$	<p>Sol.</p> $\frac{\ln^2(\ln x)}{2} + c$
<p>2.77.</p> $\int \frac{e^{2 \operatorname{tg} x}}{\cos^2 x} \, dx$	<p>Sol.</p> $\frac{1}{2} e^{2 \operatorname{tg} x} + c$
<p>2.78.</p> $\int \frac{\sin x}{\cos^2 x} \, dx$	<p>Sol.</p> $\frac{1}{\cos x} + c$
<p>2.79.</p> $\int \sin 2x \frac{x}{\sqrt{2 - \cos 2x}} \, dx$	<p>Sol.</p> $-\sqrt{2 - \cos 2x} + c$
<p>2.80.</p> $\int \sin^3 x \cos^2 x \, dx$	<p>Sol.</p> $-\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + c$
<p><i>RAONAMENT</i></p> $\int \sin^3 x \cos^2 x \, dx = \int (1 - \cos^2 x) \cos^2 x \sin x \, dx = \int (1 - t^2) t^2 \, dt = \frac{t^3}{3} - \frac{t^5}{5} + c$ $= -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + c$ $t = \cos x \quad dt = -\sin x \, dx$	

INTEGRACIÓ PER PARTS

$$\int u dv = uv - \int v du$$

3.1. $\int x \sin x \, dx$	<i>Sol.</i> $-x \cos x + \sin x + c$
3.2. $\int x \cos 3x \, dx$	<i>Sol.</i> $\frac{x \sin 3x}{3} + \frac{\cos 3x}{9} + c$
3.3. $\int x^2 \ln x \, dx$	<i>Sol.</i> $\frac{x^3 \ln x}{3} - \frac{x^3}{9} + c$
3.4. $\int x^3 e^x \, dx$	<i>Sol.</i> $x^3 e^x - 3x^2 e^x + 6x e^x + 6 e^x + c$
3.5. $\int x^2 e^{3x} \, dx$	<i>Sol.</i> $\frac{x^2 e^{3x}}{3} - \frac{2}{9} x e^{3x} + \frac{2}{27} e^{3x} + c$
RAONAMENT	
$\int x^2 e^{3x} \, dx = \frac{x^2 e^{3x}}{3} - \frac{2}{3} \int x e^{3x} \, dx = \frac{x^2 e^{3x}}{3} - \frac{2}{3} \left(\frac{x e^{3x}}{3} - \frac{1}{3} \int e^{3x} \, dx \right) =$ $\frac{x^2 e^{3x}}{3} - \frac{2}{9} x e^{3x} + \frac{2}{27} e^{3x} + c$ $\begin{cases} u = x^2 & du = 2x dx \\ dv = e^{3x} & v = \frac{1}{3} e^{3x} \end{cases} \quad \begin{cases} u = x & du = dx \\ dv = e^{3x} & v = \frac{1}{3} e^{3x} \end{cases}$	
3.6. $\int x e^x \, dx$	<i>Sol.</i> $x e^x - e^x + c$
3.7. $\int \arcsin x \, dx$	<i>Sol.</i> $x \arcsin x + \sqrt{1-x^2} + c$

3.8. $\int x \sqrt{1+2x} dx$	<i>Sol.</i> $\frac{x \sqrt{(1+2x)^3}}{3} - \frac{1}{15} \sqrt{(1+2x)^5} + c$
3.9. $\int x \operatorname{arctg} x dx$	<i>Sol.</i> $\frac{x^2 \operatorname{arc} tg x}{2} - \frac{x}{2} + \frac{\operatorname{arc} tg x}{2} + c$
3.10. $\int x^2 \sin x dx$	<i>Sol.</i> $-x^2 \cos x + 2x \sin x + 2 \cos x + c$
RAONAMENT $\int x^2 \sin x dx = -x^2 \cos x + 2 \int x \cos x dx = -x^2 \cos x + 2(x \sin x - \int \sin x dx) =$ $-x^2 \cos x + 2x \sin x + 2 \cos x + c$ $\left\{ \begin{array}{l} u = x^2 \quad du = 2x dx \\ dv = \sin x dx \quad v = -\cos x \end{array} \right\} \left\{ \begin{array}{l} u = x \quad du = dx \\ dv = \cos x dx \quad v = \sin x \end{array} \right.$	
3.11. $\int (\ln x)^2 dx$	<i>Sol.</i> $x (\ln x)^2 - 2x \ln x + 2x + c$
3.12. $\int \sin(\ln x) dx$	<i>Sol.</i> $2\sqrt{x} e^{\sqrt{x}} - 2e^{\sqrt{x}} + c$
3.13. $\int \sqrt{x} \ln x dx$	<i>Sol.</i> $\frac{2 \sqrt{x^3} \ln x}{3} - \frac{4 \sqrt{x^3}}{9} + c$
3.14. $\int \operatorname{arctg} x dx$	<i>Sol.</i> $x \operatorname{arc} tg x - \frac{1}{2} \ln 1+x^2 + c$
3.15. $\int x^2 \cos x dx$	<i>Sol.</i> $x^2 \sin x + 2x \cos x - 2 \sin x + c$
RAONAMENT $\int x^2 \cos x dx = x^2 \sin x - 2 \int x \sin x dx = x^2 \sin x - 2(-x \cos x + \int \cos x dx) =$	

$x^2 \sin x + 2x \cos x - 2 \sin x + c$ $\begin{cases} u = x^2 & du = 2x dx \\ dv = \cos x dx & v = \sin x \end{cases} \quad \begin{cases} u = x & du = dx \\ dv = \sin x dx & v = -\cos x \end{cases}$	
3.16. $\int \frac{x dx}{\sqrt{1+x}}$	Sol. $2x \sqrt{1+x} - \frac{4 \sqrt{(1+x)^3}}{3} + c$
3.17. $\int \sin(\ln x) dx$	Sol. $\frac{x \sin(\ln x) - x \cos(\ln x)}{2} + c$
3.18. $\int \frac{2x dx}{\cos^2 x}$	Sol. $2x \operatorname{tg} x + 2 \ln \cos x + c$
3.19. $\int \frac{\ln x}{\sqrt{x}} dx$	Sol. $2\sqrt{x} \ln x - 4\sqrt{x} + c$
3.20. $\int (x^2 - x) e^{-x} dx$	Sol. $-e^{-x} (x^2 - x) - e^{-x} (2x - 1) - 2e^{-x} + c$
RAONAMENT	
$\int (x^2 - x) e^{-x} dx = -(x^2 - x)e^{-x} + \int (2x - 1)e^{-x} dx = -(x^2 - x)e^{-x} - (2x - 1)e^{-x} + 2 \int e^{-x} dx = -(x^2 - x)e^{-x} - (2x - 1)e^{-x} - 2e^{-x} + c$ $\begin{cases} u = x^2 - x & du = 2x - 1 \\ dv = e^{-x} dx & v = -e^{-x} \end{cases} \quad \begin{cases} u = 2x - 1 & du = 2 dx \\ dv = e^{-x} dx & v = -e^{-x} \end{cases}$	
3.21. $\int x^3 e^{x^2} dx$	Sol. $x^2 \frac{e^{x^2}}{2} - \frac{e^{x^2}}{2} + c$
3.22. $\int \ln x dx$	Sol. $x \ln x - x + c$
3.23.	Sol.

$\int e^x \cos x \, dx$	$\frac{e^x \cos x + e^x \sin x}{2} + c$
3.24. $\int e^x \sin x \, dx$	<i>Sol.</i> $\frac{e^x \sin x - e^x \cos x}{2} + c$
3.25. $\int x e^{-3x} \, dx$	<i>Sol.</i> $-\frac{x e^{-3x}}{3} - \frac{e^{-3x}}{9} + c$
RAONAMENT	
$\int x e^{-3x} \, dx = \frac{-x e^{-3x}}{3} + \frac{1}{3} \int e^{-3x} \, dx = -\frac{x e^{-3x}}{3} - \frac{e^{-3x}}{9} + c$ $\begin{cases} u = x & du = dx \\ dv = e^{-3x} & v = \frac{-e^{-3x}}{3} \end{cases}$	
3.26. $\int \frac{x \, dx}{\cos^2 x}$	<i>Sol.</i> $x \operatorname{tg} x + \ln \cos x + c$
3.27. $\int x \cos x \, dx$	<i>Sol.</i> $x \sin x + \cos x + c$
3.28. $\int \frac{\ln x}{x^3} \, dx$	<i>Sol.</i> $-\frac{\ln x}{2x^2} - \frac{1}{4x^2} + c$
3.29. $\int x^2 \sin x \, dx$	<i>Sol.</i> $-x^2 \cos x + 2x \sin x + 2 \cos x + c$
3.30. $\int e^{-3x} \cos x \, dx$	<i>Sol.</i> $\frac{e^{-3x} \sin x}{10} - \frac{3 e^{-3x} \cos x}{10} + c$
RAONAMENT	
$I = \int e^{-3x} \cos x \, dx = e^{-3x} \sin x + 3 \int e^{-3x} \sin x \, dx = e^{-3x} \sin x + 3(-e^{-3x} \cos x - 3I)$	

$10I = e^{-3x} \sin x - 3e^{-3x} \cos x \qquad I = \frac{e^{-3x} \sin x}{10} - \frac{3e^{-3x} \cos x}{10} + c$	
$\left\{ \begin{array}{l} u = e^{-3x} \\ dv = \cos x dx \end{array} \right. \quad \left. \begin{array}{l} du = -3e^{-3x} dx \\ v = \sin x \end{array} \right\} \quad \left\{ \begin{array}{l} u = e^{-3x} \\ dv = \sin x dx \end{array} \right. \quad \left. \begin{array}{l} du = -3e^{-3x} dx \\ v = -\cos x \end{array} \right\}$	
3.31. $\int x (\ln x)^2 dx$	<i>Sol.</i> $\frac{x^2}{2} (\ln x)^2 - \frac{x^2}{2} \ln x + \frac{x^2}{4} + c$
3.32. $\int x^3 \ln x dx$	<i>Sol.</i> $\frac{x^4}{4} \ln x - \frac{x^4}{16} + c$
3.33. $\int \frac{\ln (\ln x)}{x} dx$	<i>Sol.</i> $\ln x \cdot \ln (\ln x) - \ln x + c$
3.34. $\int \frac{x dx}{\sqrt{1-x}}$	<i>Sol.</i> $-2x \sqrt{1-x} - \frac{4\sqrt{(1-x)^3}}{3} + c$
3.35. $\int (x-3) \sin x dx$	<i>Sol.</i> $-(x-3) \cos x + \sin x + c$
RAONAMENT	
$\int (x-3) \sin x dx = -(x-3) \cos x + \int \cos x dx = -(x-3) \cos x + \sin x + c$ $\left\{ \begin{array}{l} u = x-3 \\ dv = \sin x dx \end{array} \right. \quad \left. \begin{array}{l} du = dx \\ v = -\cos x \end{array} \right\}$	
3.36. $\int \ln (x + \sqrt{1+x^2}) dx$	<i>Sol.</i> $x \ln x + \sqrt{1+x^2} - \sqrt{1+x^2} + c$
3.37. $\int \frac{x \arcsin x}{\sqrt{1-x^2}} dx$	<i>Sol.</i> $-\sqrt{1-x^2} \arcsin x + x + c$
3.38.	<i>Sol.</i>

$\int x \arcsin x^2 dx$	$\frac{x^2}{2} \arcsin x^2 + \frac{1}{2} \sqrt{1-x^4} + c$
3.39. $\int \sqrt{x} (\ln x)^2 dx$	<i>Sol.</i> $\frac{2}{3} \sqrt{x^3} (\ln x)^2 - \frac{8}{9} \sqrt{x^3} (\ln x) + \frac{16}{27} \sqrt{x^3} + c$

4

INTEGRACIÓ DE FUNCIONS RACIONALS

4.1. $\int \frac{2x - 3}{x + 2} dx$	<i>Sol.</i> $2x - 7 \ln x + 2 + c$
4.2. $\int \frac{dx}{x^2 - 4}$	<i>Sol.</i> $\frac{1}{4} \ln \left \frac{x-2}{x+2} \right + c$
4.3. $\int \frac{x - 1}{x^2 + x - 6} dx$	<i>Sol.</i> $\frac{4}{5} \ln x + 3 + \frac{1}{5} \ln x - 2 + c$
4.4. $\int \frac{2 dx}{x^2 + 5x + 6}$	<i>Sol.</i> $2 \ln x + 2 - 2 \ln x + 3 + c$
4.5. $\int \frac{x + 1}{x(x - 1)^2} dx$	<i>Sol.</i> $\ln x + \ln x - 1 - \frac{2}{x - 1} + c$
RAONAMENT	

$$\frac{x+1}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} = \frac{A(x-1)^2 + Bx(x-1) + Cx}{x(x-1)^2}$$

$$x+1 = A(x-1)^2 + Bx(x-1) + Cx \begin{cases} x=1 & C=2 \\ x=0 & A=1 \\ x=2 & A+2B+2C=3 \end{cases}$$

$$\int \frac{1}{x} dx + \int \frac{1}{x-1} dx + \int \frac{2}{(x-1)^2} dx = \ln|x| + \ln|x-1| - \frac{2}{x-1} + c$$

4.6.

$$\int \frac{dx}{x^2 + 2x}$$

Sol.

$$\frac{1}{2} \ln|x| - \frac{1}{2} \ln|x+2| + c$$

4.7.

$$\int \frac{x^2 + 1}{x^2 + x - 6} dx$$

Sol.

$$x + \ln|x-2| - 2 \ln|x+3| + c$$

4.8.

$$\int \frac{x^3 - 1}{x^2 + x} dx$$

Sol.

$$\frac{x^2}{2} - x - \ln|x| + 2 \ln|x+1| + c$$

4.9.

$$\int \frac{x^2 + 1}{x^2 - 1} dx$$

Sol.

$$x - \ln|x+1| + \ln|x-1| + c$$

4.10.

$$\int \frac{dx}{x^2(x+1)}$$

Sol.

$$\ln \left| \frac{x+1}{x} \right| - \frac{1}{x} + c$$

RAONAMENT

$\frac{1}{x^2(x+1)} = \frac{A}{x+1} + \frac{B}{x} + \frac{C}{x^2} = \frac{Ax^2 + Bx(x+1) + C(x+1)}{x^2(x+1)}$ $Ax^2 + Bx(x+1) + C(x+1) = 1 \begin{cases} x = -1 & A = 1 \\ x = 0 & C = 1 \\ x = 1 & A + 2B + 2C = 1 \end{cases}$ $\int \frac{1}{x^2(x+1)} = \int \frac{1}{x+1} dx + \int \frac{-1}{x} dx + \int \frac{1}{x^2} dx = \ln x+1 - \ln x - \frac{1}{x} + c =$ $\ln \left \frac{x+1}{x} \right - \frac{1}{x} + c$	
4.11.	<i>Sol.</i>
$\int \frac{dx}{x^2 - 9}$	$\frac{1}{6} \ln \left \frac{x-3}{x+3} \right + c$
4.12.	<i>Sol.</i>
$\int \frac{x dx}{(x-1)^2 (x+1)}$	$\frac{1}{4} \ln \left \frac{x+1}{x-1} \right - \frac{1/2}{x-1} + c$
4.13.	<i>Sol.</i>
$\int \frac{6 dx}{x(x-1)(x+2)}$	$-3 \ln x + 2 \ln x-1 + \ln x+2 + c$
4.14.	<i>Sol.</i>
$\int \frac{x^2 - x + 1}{x^3 - 2x^2 + x} dx$	$\ln x - \frac{1}{x-1} + c$
4.15.	<i>Sol.</i>
$\int \frac{2x^2 + 2x - 1}{x+1} dx$	$x^2 - \ln x+1 + c$
RAONAMENT	
$\int \frac{2x^2 + 2x - 1}{x+1} dx = \int 2x dx - \int \frac{1}{x+1} dx = x^2 - \ln x+1 + c$	
4.16.	<i>Sol.</i>

$\int \frac{(2x^2 - 7x) dx}{x^3 - 3x^2 + 4}$	$\ln x+1 + \ln x-2 + \frac{2}{x-2} + c$
4.17. $\int \frac{(2x + 4) dx}{x^2 + 2x - 3}$	Sol. $\frac{1}{2} \ln x+3 + \frac{3}{2} \ln x-1 + c$
4.18. $\int \frac{dx}{(x+1)(x-2)^2(x+3)}$	Sol. $\ln x+1 - \frac{3}{x-2} - 2 \ln x+2 + c$
4.19. $\int \frac{dx}{x^3 + x^2}$	Sol. $-\ln x - \frac{1}{x} + \ln x+1 + c$
4.20. $\int \frac{(3x^2 + 2x + 5) dx}{(x-2)^2(x+1)^2}$	Sol. $-\frac{7}{3(x-2)} - \frac{2}{3(x+1)} + c$
RAONAMENT	
$\frac{3x^2 + 2x + 5}{(x-2)^2(x+1)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+1} + \frac{D}{(x+1)^2} =$ $\frac{A(x-2)(x+1)^2 + B(x+1)^2 + C(x+1)(x-2)^2 + D(x-2)^2}{(x-2)^2(x+1)^2}$	
$\left\{ \begin{array}{l} x=2 \quad 9B=21 \quad B=\frac{7}{3} \\ x=-1 \quad 9D=6 \quad D=\frac{2}{3} \\ x=0 \quad -2A+B+4C+4D=5 \quad A=0 \\ x=1 \quad -4A+4B+2C+D=10 \quad C=0 \end{array} \right.$	
$\int \frac{(3x^2 + 2x + 5) dx}{(x-2)^2(x+1)^2} = -\frac{7}{3(x-2)} - \frac{2}{3(x+1)} + c$	
4.21.	Sol.

$\int \frac{x^5 + x^4 - 8}{x^3 - 4x} dx$	$\frac{x^3}{3} + \frac{x^2}{2} + 4x + \ln \left \frac{x^2 (x-2)^5}{(x+2)^3} \right + c$
4.22. $\int \frac{x^2 - 1}{x^2 + 1} dx$	Sol. $x - 2 \operatorname{arctg} x + c$
4.23. $\int \frac{(x - 8) dx}{x^3 - 4x^2 + 4x}$	Sol. $2 \ln \left \frac{x-2}{x} \right + \frac{3}{x-2} + c$
4.24. $\int \frac{x + 1}{x^3 - 4x^2 + 5x - 2} dx$	Sol. $-3 \ln x-1 + \frac{2}{x-1} + 3 \ln x-2 + c$
4.25. $\int \frac{dx}{x^2 + 4}$	Sol. $\frac{1}{2} \operatorname{arctg} \left(\frac{x}{2} \right) + c$
RAONAMENT	
$\int \frac{dx}{x^2 + 4} = \frac{1}{4} \int \frac{dx}{1 + \left(\frac{x}{2}\right)^2} = \frac{1}{2} \int \frac{\frac{1}{2} dx}{1 + \left(\frac{x}{2}\right)^2} = \frac{1}{2} \operatorname{arctg} \left(\frac{x}{2} \right) + c$	
4.26. $\int \frac{dx}{x^3 + x^2 + x}$	Sol. $\ln x - \frac{1}{2} \ln x^2 + x + 1 - \frac{\sqrt{3}}{3} \operatorname{arctg} \left(\frac{2x+1}{\sqrt{3}} \right) + c$
4.27. $\int \frac{dx}{x^2 - 2x + 5}$	Sol. $\frac{1}{2} \operatorname{arctg} \left(\frac{x-1}{2} \right) + c$
4.28. $\int \frac{3 dx}{x^3 - 1}$	Sol. $\ln \left \frac{x-1}{\sqrt{x^2 + x + 1}} \right - \sqrt{3} \operatorname{arctg} \left(\frac{2x+1}{\sqrt{3}} \right) + c$
4.29.	Sol.

$\int \frac{5x^2 - 2x + 25}{x^3 - 6x^2 + 25x} dx$	$\ln x + 2 \ln x^2 - 6x + 25 + 4 \operatorname{arctg} \frac{x-3}{4} + c$
4.30. $\int \frac{-2x dx}{(x-1)^2(x^2+1)}$	Sol. $\frac{1}{x-1} + \operatorname{arctg} x + c$

5

ALTRES

5.1. $\int (x^3 + 3x^2 + 2x - 3) dx$	Sol. $\frac{x^4}{4} + x^3 + x^2 - 3x + c$
5.2. $\int (e^x + 3) dx$	Sol. $e^x + 3x + c$
5.3. $\int \left(e^{-x} + \sqrt[3]{x} - \frac{1}{\sqrt[3]{2x}} + \frac{1}{x^2} \right) dx$	Sol. $-e^{-x} + \frac{3 \sqrt[3]{x^4}}{4} - \frac{3}{4} \sqrt[3]{(2x)^2} - \frac{1}{x} + c$
5.4. $\int x^2 e^x dx$	Sol. $x^2 e^x - 2x e^x + 2 e^x + c$
5.5. $\int \frac{dx}{(3x+1)^4}$	Sol. $\frac{-1}{9(3x+1)^3} + c$
RAONAMENT	
$\int \frac{dx}{(3x+1)^4} = \frac{1}{3} \int t^{-4} dt = \frac{1}{3} \cdot \frac{t^{-3}}{-3} = \frac{-1}{9(3x+1)^3} + c$	
$t = 3x+1 \quad dt = 3dx$	
5.6.	Sol.

$\int \frac{3 + 2x^2}{5 + (3x + \frac{2}{3}x^3)} dx$	$\ln \left 5 + 3x + \frac{2}{3}x^3 \right + c$
5.7. $\int \frac{(2x + 1) dx}{(x^2 + x)^3}$	Sol. $\frac{-1}{2(x^2 + x)^2} + c$
5.8. $\int \frac{x}{\cos^2 x} dx$	Sol. $x \operatorname{tg} x + \ln \cos x + c$
5.9. $\int \frac{5 dx}{e^x + e^{-x}}$	Sol. $5 \operatorname{arctg} e^x + c$
5.10. $\int (1 + \operatorname{tag}^2 x^2) x dx$	Sol. $\frac{1}{2} \operatorname{tg} x^2 + c$
RAONAMENT	
$\int (1 + \operatorname{tag}^2 x^2) x dx = \int \frac{1}{2} dt = \frac{t}{2} + c = \frac{1}{2} \operatorname{tg} x^2 + c$	
$t = \operatorname{tg} x^2 \quad dt = (1 + \operatorname{tg}^2 x^2) 2x dx$	
5.11. $\int \sin^2 x dx$	Sol. $\frac{1}{2} \left(x - \frac{\sin 2x}{2} \right) + c$
5.12. $\int \operatorname{tag}^2 x dx$	Sol. $\operatorname{tg} x - x + c$
5.13. $\int (3 + \operatorname{tag}^2 x) dx$	Sol. $2x + \operatorname{tg} x + c$
5.14. $\int \frac{\sqrt{1+x}}{\sqrt{1-x}} dx$	Sol. $\operatorname{arcsin} x - \sqrt{1-x^2} + c$
5.15.	Sol.

$\int \sqrt{2+x^2} x dx$	$\frac{\sqrt{(2+x^2)^3}}{3} + c$
<i>RAONAMENT</i>	
$\int \sqrt{2+x^2} x dx = \frac{1}{2} \int t^{\frac{1}{2}} dt = \frac{1}{2} \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{1}{3} \sqrt{(2+x^2)^3} + c$	
$t = 2+x^2 \quad dt = 2x dx$	
5.16. $\int \frac{5 \cos x}{\sqrt{1+\sin x}} dx$	<i>Sol.</i> $10 \sqrt{1+\sin x} + c$
5.17. $\int \frac{e^{3x} + e^x + 1}{e^x} dx$	<i>Sol.</i> $\frac{e^{2x}}{2} + x - e^{-x} + c$
5.18. $\int \frac{dx}{\sqrt{9-x^2}}$	<i>Sol.</i> $\arcsin \left(\frac{x}{3} \right) + c$
5.19. $\int \frac{3x^3 dx}{\sqrt{x^2+1}}$	<i>Sol.</i> $3x^2 \sqrt{x^2+1} - 2\sqrt{(x^2+1)^3} + c$
5.20. $\int \frac{5^x}{3^x} dx$	<i>Sol.</i> $\left(\frac{5}{3} \right)^x \cdot \frac{1}{\ln(5/3)} + c$
<i>RAONAMENT</i>	
$\int \frac{5^x}{3^x} dx = \int \left(\frac{5}{3} \right)^x dx = \frac{1}{\ln \frac{5}{3}} \int dt = \frac{1}{\ln \frac{5}{3}} t + c = c + \frac{1}{\ln \frac{5}{3}} \left(\frac{5}{3} \right)^x$	
$t = \left(\frac{5}{3} \right)^x \quad dt = \left(\frac{5}{3} \right)^x \ln \left(\frac{5}{3} \right) dx$	
5.21.	<i>Sol.</i> $-\frac{\ln x}{x} - \frac{1}{x} + c$

$\int \frac{\ln x}{x^2} dx$	
5.22. $\int \left(\frac{6x^2}{\sin^2 x^3} + \frac{4}{\cos^2 4x} \right) dx$	<i>Sol.</i> $-2 \cotg(x^3) + \tg(4x) + c$
5.23. $\int \frac{dx}{e^{2x+1}}$	<i>Sol.</i> $-\frac{1}{2} e^{-2x-1} + c$
5.24. $\int \frac{dx}{x^2 + 4}$	<i>Sol.</i> $\frac{1}{2} \operatorname{arctg} \left(\frac{x}{2} \right) + c$
5.25. $\int \frac{x^2 dx}{x^3 + 4}$	<i>Sol.</i> $\frac{1}{3} \ln x^3 + 4 + c$
RAONAMENT	
$\int \frac{x^2 dx}{x^3 + 4} = \frac{1}{3} \int \frac{dt}{t} = \frac{1}{3} \ln t + c = \frac{1}{3} \ln x^3 + 4 + c$ $t = x^3 + 4 \quad dt = 3x^2 dx$	
5.26. $\int \frac{e^{3x}}{1 + e^{6x}} dx$	<i>Sol.</i> $\frac{1}{3} \operatorname{arctg} (e^{3x}) + c$
5.27. $\int e^{-5x^2} (-5x) dx$	<i>Sol.</i> $\frac{1}{2} e^{-5x^2} + c$
5.28. $\int \frac{\tg x}{\cos^2 x} dx$	<i>Sol.</i> $\frac{\tg^2 x}{2} + c$
5.29. $\int \tg x dx$	<i>Sol.</i> $-\ln \cos x + c$
5.30. $\int (\cos 5x - 3 \sin 2x) dx$	<i>Sol.</i>

	$\frac{\sin 5x}{5} + \frac{3 \cos 2x}{2} + c$
RAONAMENT	
$\int (\cos 5x - 3 \sin 2x) dx = \int \cos 5x dx - 3 \int \sin 2x dx = \frac{1}{5} \sin 5x + \frac{3}{2} \cos 2x + c$	
5.31. $\int \frac{x dx}{1 + (x^2 + 3)^2}$	Sol. $\frac{1}{2} \operatorname{arctg}(x^2 + 3) + c$
5.32. $\int \frac{\ln x}{x} dx$	Sol. $\frac{\ln^2 x }{2} + c$
5.33. $\int (x - e^x \cos x) dx$	Sol. $\frac{x^2}{2} - \frac{e^x \cos x + e^x \sin x}{2} + c$
5.34. $\int \frac{dx}{1 - \sin x}$	Sol. $\frac{2}{1 - \operatorname{tg}(x/2)} + c$
5.35. $\int e^{\sin x} \cos x dx$	Sol. $e^{\sin x} + c$
RAONAMENT	
$\int e^{\sin x} \cos x dx = \int e^t dt = e^t + c = e^{\sin x} + c$ $t = \sin x \quad dt = \cos x dx$	
5.36. $\int \sin^3 x \cos^3 x dx$	Sol. $\frac{\sin^4 x}{4} - \frac{\sin^6 x}{6} + c$
5.37. $\int x \cos(1 + x^2) dx$	Sol. $\frac{1}{2} \sin(1 + x^2) + c$
5.38. $\int \frac{dx}{\sqrt{x} (1 + \sqrt{x})}$	Sol. $2 \ln 1 + \sqrt{x} + c$
5.39.	Sol.

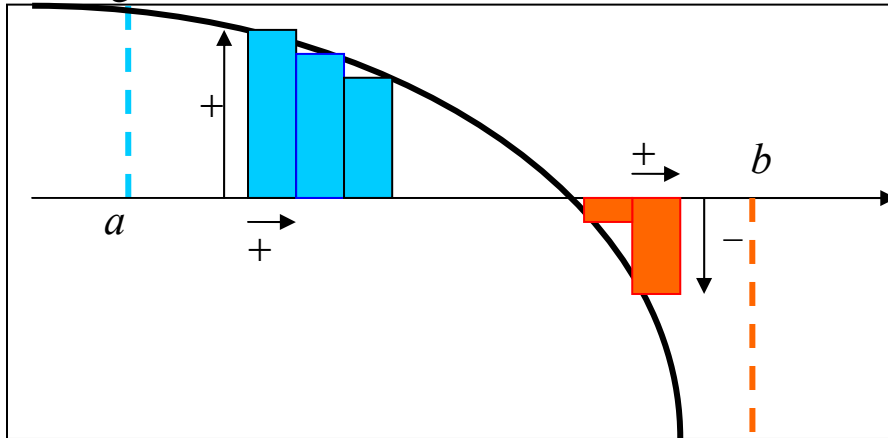
$\int \frac{x+9}{x^2-9} dx$	$2 \ln x-3 - \ln x+3 + c$
5.40. $\int \frac{5e^x}{2+e^x} dx$	Sol. $5 \ln 2+e^x + c$
RAONAMENT $\int \frac{5e^x}{2+e^x} dx = \int \frac{5dt}{2+t} = 5 \ln 2+t + c = 5 \ln 2+e^x + c$ $t = e^x \quad dt = e^x dx$	
5.41. $\int \frac{x - \sqrt{x}}{\sqrt{x} - \sqrt[3]{x}} dx$	Sol. $\frac{2 \sqrt[6]{x^9}}{3} + \frac{3 \sqrt[3]{x^4}}{4} + \frac{6 \sqrt[6]{x^7}}{7} + c$
5.42. $\int \frac{dx}{1 - \sin^2 x}$	Sol. $\operatorname{tg}(x) + c$
5.43. $\int \frac{x dx}{x + \sqrt{x}}$	Sol. $x - \sqrt{x} - 2 \ln \sqrt{x} + 1 + c$
5.44. $\int \frac{\operatorname{tg}^3 x}{\cos^2 x} dx$	Sol. $\frac{\operatorname{tg}^4 x}{4} + c$
5.45. $\int \frac{e^{\operatorname{tg} x}}{\cos^2 x} dx$	Sol. $e^{\operatorname{tg} x} + c$
RAONAMENT $\int \frac{e^{\operatorname{tg} x}}{\cos^2 x} dx = \int e^t dt = e^t + c = e^{\operatorname{tg} x} + c$ $t = \operatorname{tg} x \quad dt = \frac{1}{\cos^2 x} dx$	
5.46.	Sol.

$\int \frac{2x}{9 + 5x^2} dx$	$\frac{1}{5} \ln 9 + 5x^2 + c$
5.47. $\int \frac{2x^3 + x^2 + 3x + 1}{x + 1} dx$	<i>Sol.</i> $\frac{2x^3}{3} - \frac{x^2}{2} + 4x - 3 \ln x + 1 + c$
5.48. $\int \frac{1 + \sin^2 x}{\sin x \cos x} dx$	<i>Sol.</i> $\ln \sin x - 2 \ln \cos x + c$
5.49. $\int \frac{\cos x}{\sin^3 x} dx$	<i>Sol.</i> $\frac{-1}{2 \sin^2 x} + c$
5.50. $\int \frac{2x dx}{\sqrt{1 - x^4}} dx$	<i>Sol.</i> $\arcsin(x^2) + c$
RAONAMENT	
$\int \frac{2x dx}{\sqrt{1 - x^4}} dx = \int \frac{dt}{\sqrt{1 - t^2}} = \arcsin t + c = \arcsin x^2 + c$ $t = x^2 \quad dt = 2x dx$	

INTEGRAL DEFINIDA .CÀLCUL DE L'ÀREA D'UN RECINTE

INTEGRAL DE RIEMANN $\int_a^b f(x)dx$

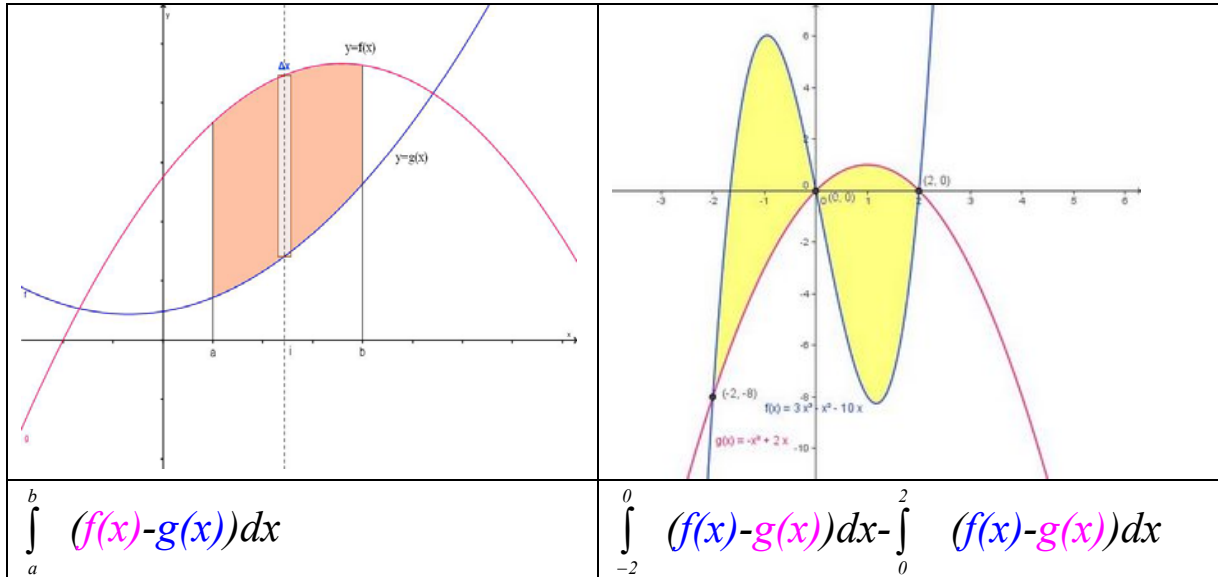
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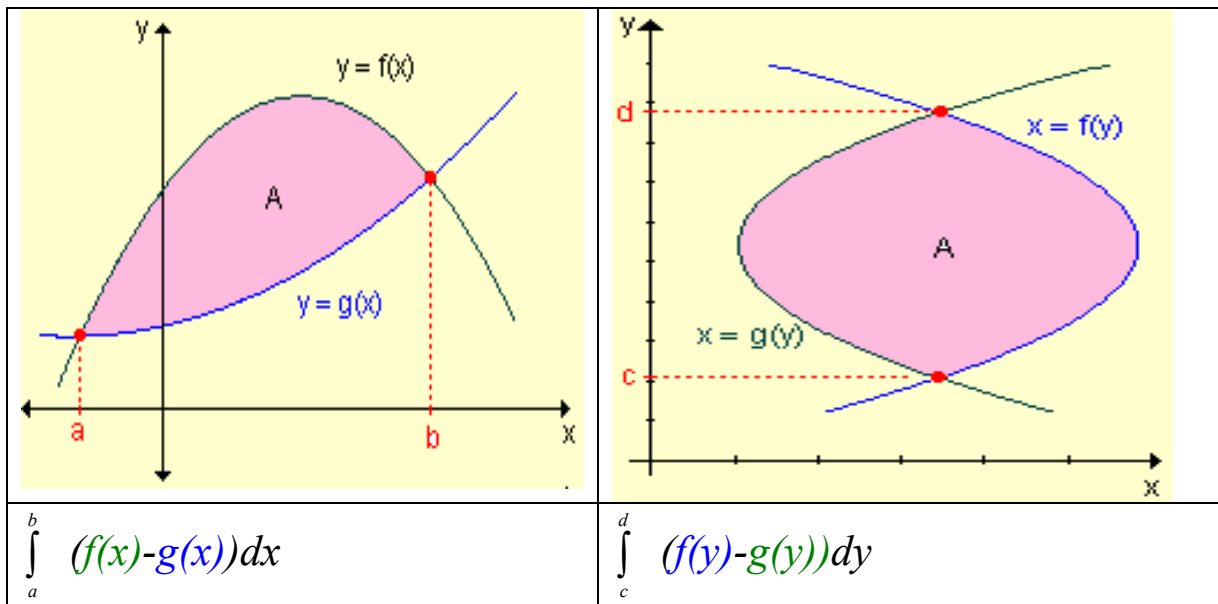


REGLA DE BARROW

Si $\int f(x)dx = F(x) + c$ aleshores $\int_a^b f(x)dx = F(b) - F(a)$

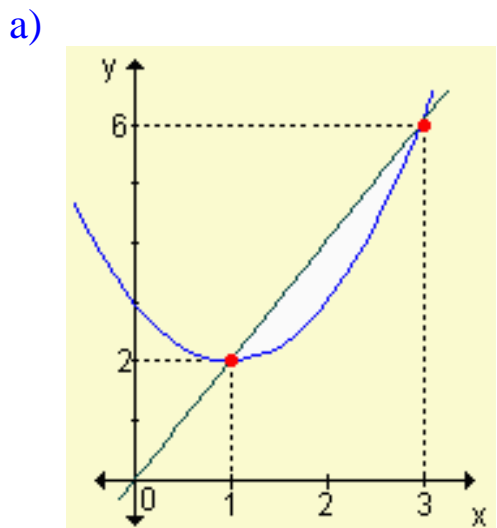
ÀREA DEL RECINTE TANCAT PER DUES FUNCIONS:





EXEMPLES

Calculeu l'àrea que tanquen les dues funcions



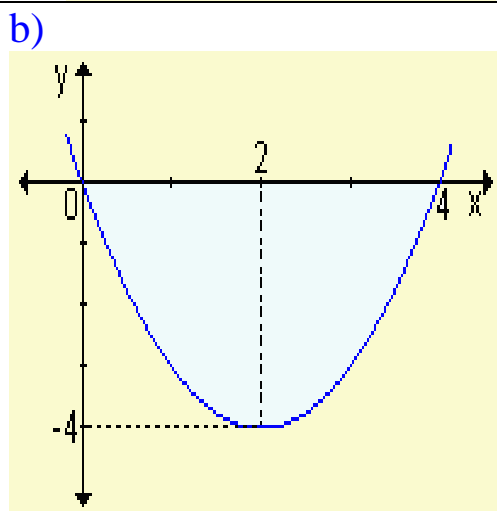
a) raonament

$$\begin{cases} f(x) = 2x \\ g(x) = (x-1)^2 + 2 \end{cases}$$

$$A = \int_{x=1}^{x=3} (f(x) - g(x)) dx =$$

$$\int_1^3 (-x^2 + 4x - 3) dx = \left[-\frac{x^3}{3} + 2x^2 - 3x \right]_1^3 =$$

$$0 - \left(-\frac{4}{3}\right) = \frac{4}{3} \text{ u.a.}$$



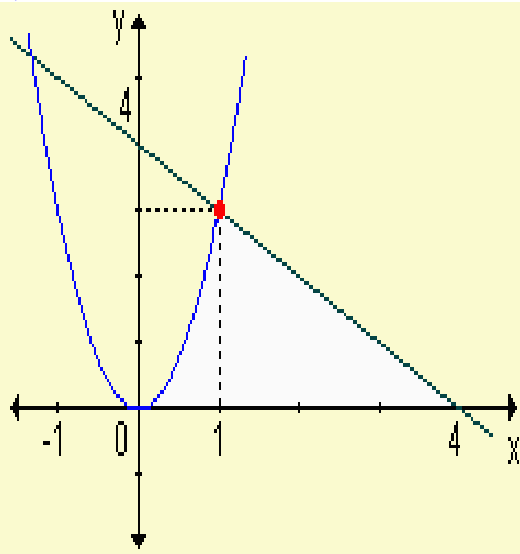
b) raonament

$$\begin{cases} f(x) = 0 \\ g(x) = (x-2)^2 - 4 \end{cases}$$

$$A = \int_{x=0}^{x=4} (f(x) - g(x)) dx = \int_0^4 (-x^2 + 4x) dx$$

$$= \left[-\frac{x^3}{3} + 2x^2 \right]_0^4 = -\frac{64}{3} + 32 = \frac{32}{3} \text{ u.a.}$$

c) raonament



c) raonament

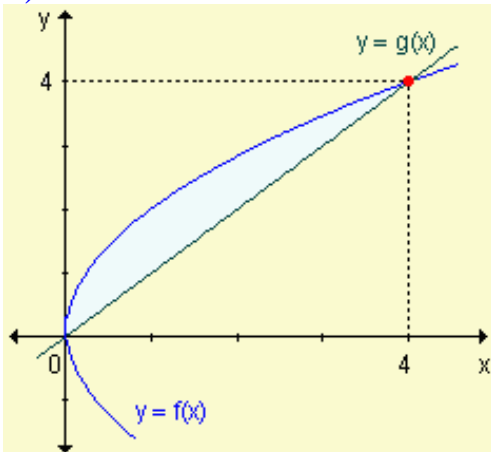
$$\begin{cases} y = g(x) = 3x^2 \\ y = f(x) = -x + 4 \end{cases} \quad \begin{cases} x = g(y) = \sqrt{\frac{y}{3}} \\ x = f(y) = -y + 4 \end{cases}$$

$$A = \int_{y=0}^{y=3} (f(y) - g(y)) dy = \int_0^1 g(x) dx +$$

$$\int_1^4 f(x) dx = \int_1^1 3x^2 dx + \int_1^4 (-x + 4) dx$$

$$= 1 + \frac{9}{2} = \frac{11}{2} \text{ u.a.}$$

d) raonament



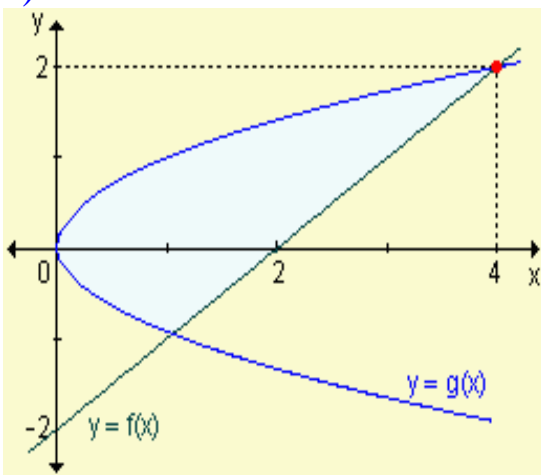
d) raonament

$$\begin{cases} f(x) = 2\sqrt{x} \\ g(x) = x \end{cases}$$

$$A = \int_{x=0}^{x=4} (f(x) - g(x)) dx = \int_0^4 (2\sqrt{x} - x) dx =$$

$$= \left[\frac{4}{3} x^{\frac{3}{2}} - \frac{1}{2} x^2 \right]_0^4 = \frac{8}{3} - 0 = \frac{8}{3}$$

e)



e) raonament

$$\begin{cases} y = f(x) = x - 2 \\ y = \pm\sqrt{x} \end{cases} \quad \begin{cases} x = f(y) = y + 2 \\ x = g(y) = y^2 \end{cases}$$

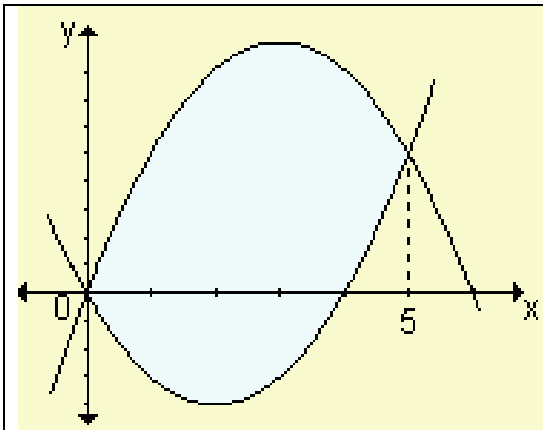
$$y^2 = y + 2 \Rightarrow \begin{cases} y = -1 \\ y = 2 \end{cases}$$

$$A = \int_{y=-1}^{y=2} (f(y) - g(y)) dy = \left[\frac{y^2}{2} + 2y - \frac{y^3}{3} \right]_{-1}^2$$

$$= \left(\frac{10}{3} + \frac{7}{6} \right) = \frac{27}{6} \text{ u.a.}$$

f)

f) raonament



Punts de tall (0,0) i (5,2)

$$\begin{cases} f(x) = -\frac{2}{5}x(x-6) \\ g(x) = \frac{2}{5}x(x-4) \end{cases}$$

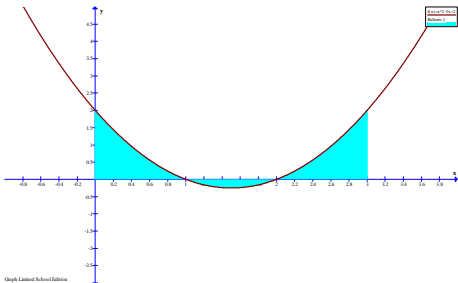
$$A = \int_{x=0}^{x=5} (f(x) - g(x)) dx = \frac{4}{5} \int_0^5 x dx = \frac{2}{5} [x^2]_0^5 = \frac{2}{5} (25 - 0) = 10 u.a.$$

EXERCICIS

1

Calculeu l'àrea del recinte limitat per la funció: $f(x) = x^2 - 3x + 2$ l'eix OX, ($y = 0$) i les rectes ($x=0$) i ($x=3$).

RAONAMENT



Punts de tall amb l'eix horitzontal ($y=0$)

$$x^2 - 3x + 2 = 0 \quad \Rightarrow \quad \begin{cases} x = 1 \\ x = 2 \end{cases} \Rightarrow \begin{cases} (1,0) \\ (2,0) \end{cases}$$

contínua i definida positiva a l'interval $(-\infty, 1) \cup (2, \infty)$

contínua i definida negativa a l'interval $(1, 2)$

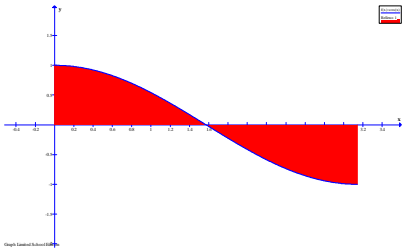
$$F(x) = \int f(x) dx = \frac{x^3}{3} - 3\frac{x^2}{2} + 2x \Rightarrow \begin{cases} F(0) = 0 & F(1) = 5/6 \\ F(2) = 2/3 & F(3) = 3/2 \end{cases}$$

$$A = \int_0^1 f(x) dx - \int_1^2 f(x) dx + \int_2^3 f(x) dx = F(1) - F(0) - F(2) + F(1) + F(3) - F(2) = 2F(1) - 2F(2) + F(3) - F(0) = \frac{5}{3} - \frac{4}{3} + \frac{3}{2} - 0 = \frac{11}{6} u.a.$$

2

Calculeu l'àrea del recinte limitat per la funció $f(x) = \cos x$, l'eix OX ($y=0$) i les rectes ($x=0$) i ($x=\pi$).

RAONAMENT



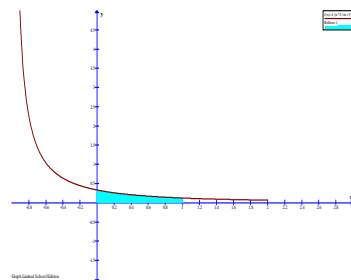
$$\int f(x)dx = \int \cos x dx = \sin x$$

$$A = \int_0^{\pi/2} f(x)dx - \int_{\pi/2}^{\pi} f(x)dx = 2 \int_0^{\pi/2} \cos x dx = 2[1-0] = 2 \text{ u.a.}$$

3

Calculeu l'àrea del recinte limitat per les rectes ($x=0$) i ($x=1$) i la funció $y = \frac{1}{(x+1)(x+3)}$.

RAONAMENT



$f(x)$ contínua, derivable i definida positiva a l'interval $[0,1]$.

$$\frac{1}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3} \Rightarrow A(x+3) + B(x+1) = 1 \Rightarrow$$

$$\begin{cases} \text{si } x = -1 & 2A = 1 & A = \frac{1}{2} \\ \text{si } x = -3 & -2B = 1 & B = -\frac{1}{2} \end{cases} \Rightarrow$$

$$\int f(x) dx = \frac{1}{2} \ln(x+1) - \frac{1}{2} \ln(x+3) =$$

$$= \ln \sqrt{\frac{x+1}{x+3}} \quad A = \int_0^1 f(x) dx = \ln \sqrt{\frac{1+1}{1+3}} - \ln \sqrt{\frac{0+1}{0+3}} = \ln \sqrt{\frac{2}{3}}$$

4

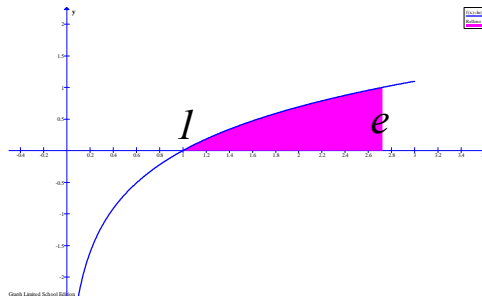
Trobeu l'àrea que tanquen les corbes: $y = \ln x$, $y = 0$ i $x = e$.

RAONAMENT

Si $y = 0$, $x = 1$

$$\int \ln x dx = x \ln x - x$$

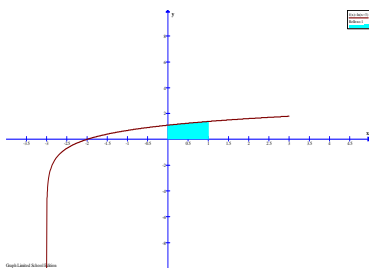
$$A = \int_1^e \ln x dx = (e - e) - (-1) = 1 \text{ u. a.}$$



5

Calculeu l'àrea del recinte limitat per la funció: $y = \ln(x+3)$, l'eix OX' ($y=0$) i les rectes $(x=0)$ i $(x=1)$.

RAONAMENT



$f(x)=\ln(x+3)$ contínua i definida positiva en $[0,1]$.

$$\int \ln(x+3) dx = x \ln(x+3) - \int \frac{x}{x+3} dx = (x+3) \ln(x+3) - x$$

$$u = \ln(x+3) \quad du = \frac{dx}{x+3}$$

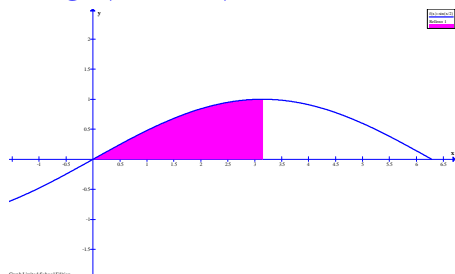
$$dv = dx \quad v = x$$

$$A = \int_0^1 \ln(x+3) dx = (4 \ln 4 - 1) - (3 \ln 3 - 0) = 4 \ln 4 - 3 \ln 3 - 1$$

6

Calculeu l'àrea del recinte limitat per la funció: $f(x) = \sin \frac{x}{2}$, l'eix OX ($y=0$) i les rectes $(x=0)$ i $(x=\pi)$.

RAONAMENT



La funció $f(x)$ és contínua i definida positiva a l'interval $(0, \pi)$

$$\int f(x) dx = \int \sin \frac{x}{2} dx = 2 \int \frac{1}{2} \sin \frac{x}{2} dx = -2 \cos \frac{x}{2} + c$$

$$A = \int_0^{\pi} f(x) dx = (0) - (-2) = 2 \text{ u.a.}$$

7

Calculeu l'àrea del recinte limitat per les funcions: $f(x) = 4x - x^2$ i $g(x) = x^2 + 2x$.

RAONAMENT

Punts de tall d'ambdues funcions:

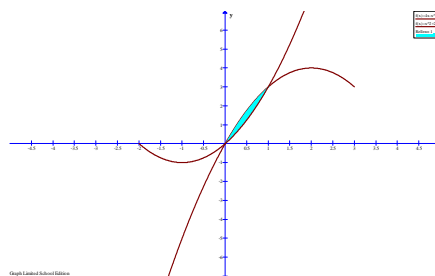
$$\begin{cases} y = 4x - x^2 \\ y = x^2 + 2x \end{cases} \Rightarrow \begin{cases} x = 0 & y = 0 \\ x = 1 & y = 3 \end{cases} \Rightarrow \begin{cases} (0,0) \\ (1,3) \end{cases}$$

interval $[0,1]$

$f(x)$ i $g(x)$ són contínues i positives a l'interval $[0,1]$ amb $f(x) > g(x)$

$$A = \int_0^1 [f(x) - g(x)] dx = \int_0^1 [-2x^2 + 2x] dx = \left(-\frac{2x^3}{3} + x^2 \right)_0^1 = -\frac{2}{3} + 1 = \frac{1}{3}$$

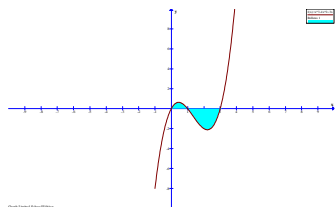
$A = 1/3$ u.a.



8

Calculeu l'àrea del recinte limitat per la funció: $f(x) = x^3 - 4x^2 + 3x$ i l'eix OX.

RAONAMENT



Punts de tall amb l'eix OX $\begin{cases} y = x^3 - 4x^2 + 3x \\ y = 0 \end{cases} \Rightarrow x=0, x=1, x=3$

interval $(0,1) \cup (1,3)$ en $(0,1)$ $f(x) > 0$ i en $(1,3)$ $f(x) < 0$

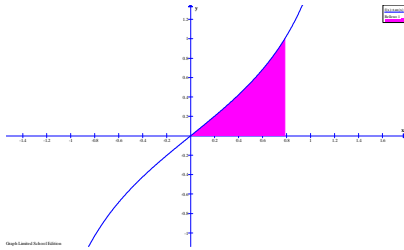
$$F(x) = \int (x^3 - 4x^2 + 3x) dx = \frac{x^4}{4} - 4\frac{x^3}{3} + 3\frac{x^2}{2} \rightarrow \begin{cases} F(0) = 0 \\ F(1) = 5/12 \\ F(3) = -9/4 \end{cases}$$

$$A = \int_0^1 f(x) - \int_1^3 f(x) = 2F(1) - F(3) = \frac{37}{12}$$

9

Àrea que delimita la funció $y = \operatorname{tg}(x)$, amb l'eix OX i la recta $x = \pi/4$.

RAONAMENT



$$\int \operatorname{tg} x dx = -\ln(\cos x) \quad \text{si } x = 0 \quad \operatorname{tg} x = 0 \quad \text{punt de tall } (0, 0)$$

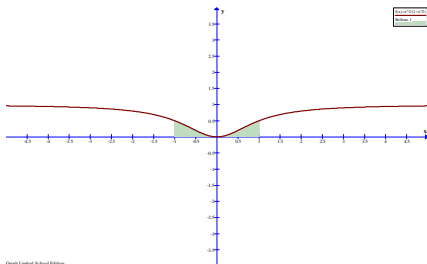
$$A = \int_0^{\pi/4} \operatorname{tg} x dx = -\ln(\cos(\pi/4)) + \ln(\cos(0)) = -\ln\left(\frac{1}{\sqrt{2}}\right) = \ln\sqrt{2} \text{ u. a.}$$

10

Calculeu l'àrea del recinte limitat per la funció

$f(x) = \frac{x^2}{1+x^2}$, l'eix OX ($y=0$) i les rectes verticals ($x=-1$) i ($x=1$).

RAONAMENT



$f(x)$ contínua a tot \mathbb{R} i toca a l'eix horitzontal en $x=0$, i en tot $x \neq 0$ $f(x)$ és positiva.

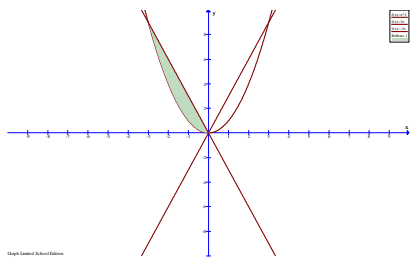
$$\int f(x) dx = \int \left(1 - \frac{1}{1+x^2}\right) dx = x - \operatorname{arctg} x$$

$$A = \int_{-1}^1 \frac{x^2}{1+x^2} dx = \left(1 - \frac{\pi}{4}\right) - \left(-1 + \frac{\pi}{4}\right) = 2 - \frac{\pi}{2} \text{ u.a.}$$

11

Calcular el valor d' m per tal que l'àrea del recinte limitat per la corba $f(x) = x^2$ i la recta $g(x) = mx$ sigui $\frac{9}{2}$ u. a.

RAONAMENT



Punts de tall d'ambdues funcions,

$$\begin{cases} y = x^2 \\ y = mx \end{cases} \Rightarrow \begin{cases} x = 0 & y = 0 \\ x = m & y = m^2 \end{cases}$$

Si $m > 0$ interval $(0, m)$ amb $g(x) > f(x)$

aleshores:

$$A = \int_0^m (g(x) - f(x)) dx = \int_0^m (mx - x^2) dx = \left(\frac{m^2}{2} - \frac{m^3}{3}\right) - 0$$

$$\text{Si } \frac{m^3}{6} = \frac{9}{2} \Rightarrow m^3 = 27 \Rightarrow m = 3$$

Si $m < 0$ interval $(m, 0)$ amb $g(x) > f(x)$ aleshores:

$$A = \int_m^0 (g(x) - f(x)) dx = \int_m^0 (mx - x^2) dx = 0 - \left(\frac{m^2}{2} - \frac{m^3}{3}\right)$$

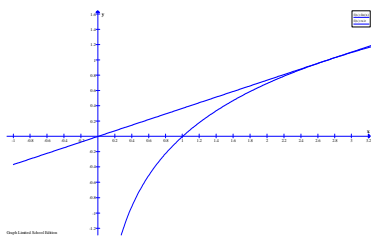
$$\text{Si } -\frac{m^3}{6} = \frac{9}{2} \Rightarrow m^3 = -27 \Rightarrow m = -3$$

Solucions $m = \pm 3$

12

Calculeu l'àrea compresa per la funció $y = \ln x$, l'eix OX ($y=0$) i la recta tangent a la funció en el punt d'abscissa $x=e$.

RAONAMENT



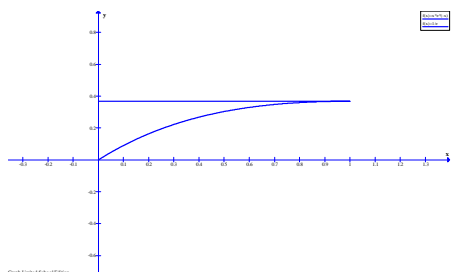
Punt de tangència $(e, 1)$ $m = y'(e) = 1/e$ recta tangent $y = (1/e)x$
 $(0, 0)$ punt de tall de la tangent i l'eix horitzontal $\int \ln x dx = x \ln x - x$

$$A = e/2 - \frac{e}{2} - \int_1^e \ln x dx = \frac{e}{2} - [0 - (-1)] = \frac{e}{2} - 1 \text{ u. a.}$$

13

Àrea limitada per $f(x) = xe^{-x} = \frac{x}{e^x}$ i les rectes $(x=0)$ i $(y = l'$ ordenada en el seu punt màxim).

RAONAMENT



ordenada del punt màxim:

$$f'(x) = \frac{e^x(1-x)}{e^{2x}} = \frac{1-x}{e^x} = 0 \Rightarrow x = 1$$

$$\Rightarrow y = \frac{1}{e} \Rightarrow P(1, \frac{1}{e})$$

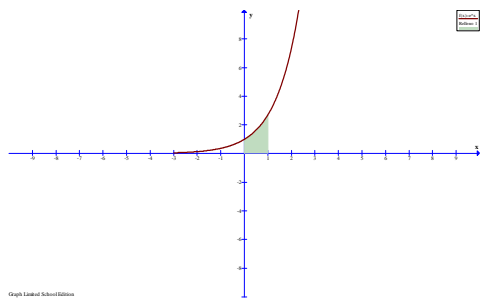
$$\int xe^{-x} dx = -xe^{-x} + \int e^{-x} dx = -(x+1)e^{-x}$$

$$A = \frac{1}{e} - \int_0^1 f(x) dx = \frac{1}{e} + \left[\frac{x+1}{e^x} \right]_0^1 = \frac{3}{e} - 1 \text{ u. a.}$$

14

Àrea limitada per la funció $y = e^x$, la seva recta tangent en el punt d'abscissa $x=1$ i l'eix vertical $(x=0)$.

RAONAMENT



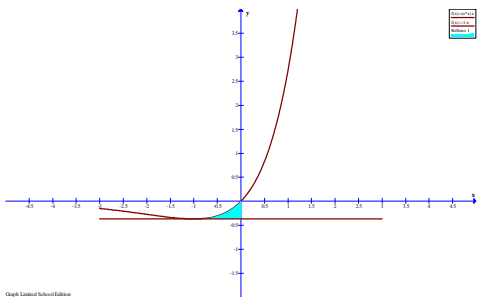
Punt de tangència, $x = 1 \Rightarrow y = e$
 punt $A(1, e)$
 Equació de la recta tangent:
 $m = y'(1) = e$ i passa pel punt $A(1, e)$
 aleshores $y - e = e(x - 1) \Rightarrow$
 $y = ex$

$$A = \int_0^1 e^x dx - \frac{e}{2} = (e^1 - e^0) - \frac{e}{2} = \frac{e}{2} - 1 \quad \text{u.a.}$$

15

Àrea limitada per la corba $y = xe^x$, l'eix $OY(x=0)$ i la recta ($y=l$ 'ordenada corresponent al punt mínim de la funció).

RAONAMENT



punt mínim $y' = e^x(1+x) = 0 \Rightarrow x = -1$ punt $A(-1, \frac{-1}{e})$

$$\text{recinte} \begin{cases} x = 0 \\ y = -\frac{1}{e} \\ y = xe^x \end{cases} \quad F(x) = \int xe^x dx = e^x(x-1)$$

$$A = \frac{1}{e} + \int_{-1}^0 xe^x dx = \frac{1}{e} + [(-1) - (-2e^{-1})] = \frac{3}{e} - 1 \quad \text{u.a.}$$

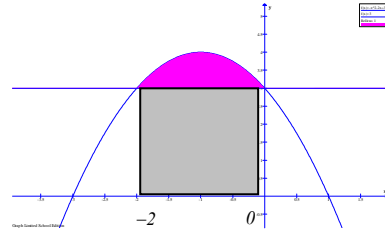
16

Àrea limitada per la corba: $y = -x^2 - 2x + 3$ i la recta ($y=3$).

RAONAMENT

Punts de tall:

$$\begin{cases} y = -x^2 - 2x + 3 \\ y = 3 \end{cases} \begin{cases} x = -2 & y = 3 \\ x = 0 & y = 3 \end{cases}$$



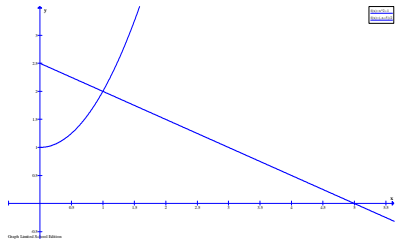
$$\int y dx = \int (-x^2 - 2x + 3) dx = -\frac{x^3}{3} - x^2 + 3x + c$$

$$A = \int_{-2}^0 y dx - 6 = (22/3) - 6 = 4/3 \text{ u. a.}$$

17

Trobeu l'àrea que delimita la funció $f(x) = x^2 + 1$ amb la seva recta normal en $x=1$ i els eixos de coordenades.

RAONAMENT



Punt de contacte $(1, 2)$ pendent de la normal $m = \frac{-1}{y'(1)} = -\frac{1}{2}$

Equació de la recta normal $y = -\frac{1}{2}x + \frac{5}{2}$

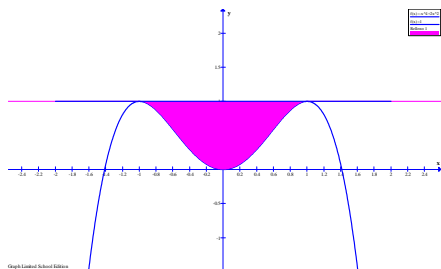
Punt de tall de la normal amb l'eix horitzontal $(5, 0)$


$$\int f(x) dx = \int (x^2 + 1) dx = \frac{x^3}{3} + x + c \quad A = \int_0^1 f(x) dx + \frac{4 \cdot 2}{2} = \frac{4}{3} + 4 \text{ u. a.}$$

18

Trobeu l'àrea que tanquen les corbes: $f(x) = -x^4 + 2x^2$ i $g(x) = 1$.

RAONAMENT



 Punts de tall d'ambdues funcions,

$$\begin{cases} y = 1 \\ y = -x^4 + 2x^2 \end{cases} \Rightarrow \begin{cases} x = \pm 1 \\ y = 1 \end{cases}$$

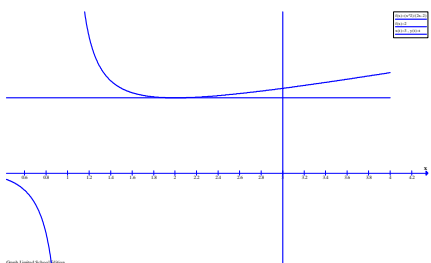
$$\int f(x) dx = \frac{-x^5}{5} + \frac{2x^3}{3} + c$$

$$B = 2 \int_0^1 y dx = 2 \left(-\frac{1}{5} + \frac{2}{3} \right) = \frac{14}{15} \text{ u. a.} \quad A = 2 - B = 2 - \frac{14}{15} = \frac{16}{15} \text{ u. a.}$$

19

Trobeu l'àrea compresa entre la funció $f(x) = \frac{x^2}{2x-2}$ i les rectes: $x=3$ i $g(x)=2$.

RAONAMENT



 Punts de tall entre $f(x)$ i $g(x)$,

$$\begin{cases} y = \frac{x^2}{2x-2} \\ y = 2 \end{cases} \Rightarrow \begin{cases} x = 2 \\ y = 2 \end{cases} \text{ punt } (2,2)$$

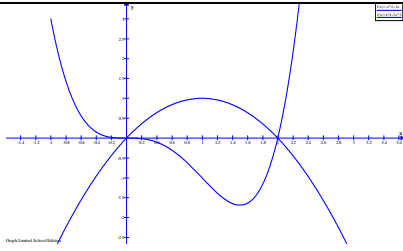
$$\int f(x) dx = \frac{1}{2} [x^2/2 + x + \ln(x-1)]$$

$$A = \int_2^3 f(x) dx - 2 = \frac{1}{2} [(9/2 + 3 + \ln 2) - (4)] - 2 = -\frac{1}{4} + \frac{1}{2} \ln 2 \text{ u. a.}$$

20

Àrea que tanquen les corbes: $f(x) = -x^2 + 2x$ i $g(x) = x^3(x-2)$.

RAONAMENT



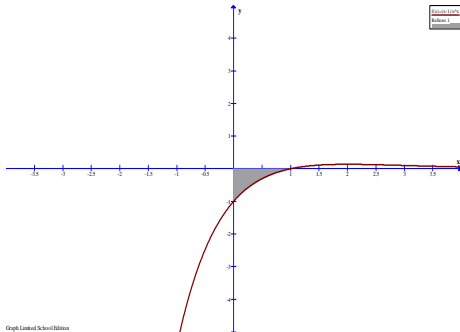
punts de tall: $x=0$ $x=2$

$$\int (g - f) dx = \frac{x^5}{5} - \frac{x^4}{2} + \frac{x^3}{3} - x^2 \quad A = \left| \int_0^2 (g - f) dx \right| = \frac{32}{5} - 8 + \frac{8}{3} - 4$$

21

Àrea que tanca la funció: $f(x) = (x-1)e^{-x} = \frac{x-1}{e^x}$ i els eixos de coordenades, ($y=0$) i ($x=0$)

RAONAMENT



$$y = 0 \Rightarrow x - 1 = 0 \Rightarrow x = 1 \quad A = \int_0^1 -\frac{x-1}{e^x} dx = -(F(1) - F(0)) = \frac{1}{e} \text{ u.a.}$$

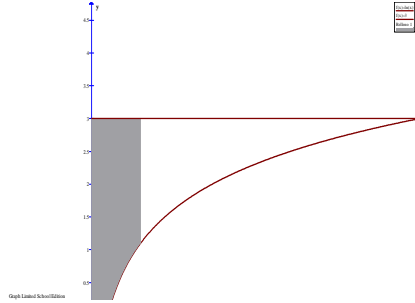
$$F(x) = \int (x-1)e^{-x} dx = -(x-1)e^{-x} + \int e^{-x} dx = -(x-1)e^{-x} - e^{-x} = -xe^{-x}$$

$$\begin{cases} u = x - 1 & du = dx \\ dv = e^{-x} dx & v = -e^{-x} \end{cases}$$

22

Trobeu l'àrea que determinen les funcions $y = \ln x$, $y = 3$ i els eixos de coordenades ($x=0, y=0$).

RAONAMENT



$$\begin{cases} f(x) = 3 \\ g(x) = \ln x \end{cases} \text{ punts de tall } \ln x = 3 \Rightarrow x = e^3$$

$$A = \int_0^{e^3} f(x) - g(x) = \int_0^{e^3} (3 - \ln x) dx = [4x - x \ln x]_0^{e^3} = (4e^3 - 3e^3) - 0 = \boxed{e^3}$$

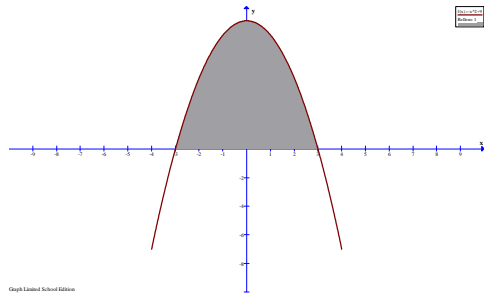
$$F(x) = \int \ln x dx = x \ln x - \int 1 dx = x \ln x - x$$

$$\begin{cases} u = \ln x & du = \frac{1}{x} dx \\ dv = dx & v = x \end{cases}$$

23

Trobeu el valor d' a per tal que l'àrea que delimita la corba $y = -x^2 + a$ amb l'eix OX ($y=0$), sigui igual a: 18 u.a.

RAONAMENT



$$\text{Tall amb l'eix } OX \quad \begin{cases} y = -x^2 + a \\ y = 0 \end{cases} \Rightarrow x = \pm\sqrt{a}$$

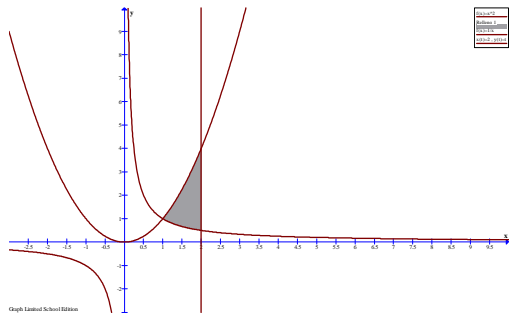
$$A = 2 \int_0^{\sqrt{a}} (-x^2 + a) dx = 2 \left[\frac{-x^3}{3} + ax \right]_0^{\sqrt{a}} = \left(\frac{-a\sqrt{a}}{3} + a\sqrt{a} \right) - 0 = \frac{2}{3} a\sqrt{a}$$

$$\text{Si } \frac{2}{3} a\sqrt{a} = 18 \Rightarrow a\sqrt{a} = 27 \Rightarrow a = 9$$

24

Calculeu l'àrea compresa per les corbes $y = x^2$, $y = \frac{1}{x}$ i la recta $x=2$.

RAONAMENT



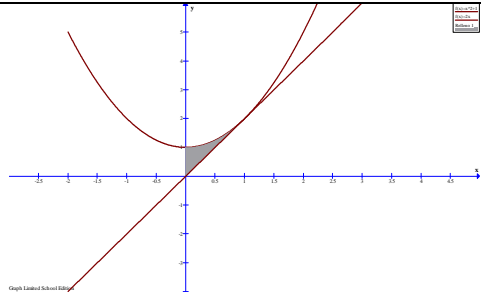
Punt de tall d'ambdues funcions
$$\begin{cases} y = x^2 \\ y = \frac{1}{x} \end{cases} \Rightarrow x^3 = 1 \Rightarrow x = 1$$

$$A = \int_1^2 \left(x^2 - \frac{1}{x} \right) dx = \left[\frac{x^3}{3} - \ln x \right]_1^2 = \left(\frac{8}{3} - \ln 2 \right) - \left(\frac{1}{3} - 0 \right) = \frac{7}{3} - \ln 2 \text{ u.a.}$$

25

Trobeu l'àrea que delimita $y = x^2 + 1$, la seva tangent en $x=1$ i l'eix vertical ($x=0$).

RAONAMENT



Recta tangent en el punt (1,2)

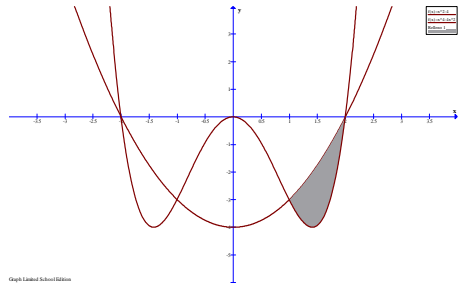
$$\begin{cases} y - 2 = m(x - 1) \\ m = y'(1) = 2(1) = 2 \end{cases} \Rightarrow y = 2x$$

$$A = \int_0^1 [(x^2 + 1) - 2x] dx = \left[\frac{x^3}{3} - x^2 + x \right]_0^1 = \left(\frac{8}{3} - 2 \right) - \left(\frac{1}{3} \right) = \frac{1}{3} u.a.$$

26

Trobeu l'àrea que tanquen les corbes: $y = x^4 - 4x^2$ i $y = x^2 - 4$.

RAONAMENT



Punts de tall entre les dues funcions

$$\begin{cases} y = x^4 - 4x^2 \\ y = x^2 - 4 \end{cases} \Rightarrow x^4 - 5x^2 + 4 = 0 \Rightarrow \begin{cases} x^2 = 1 & x = \pm 1 \\ x^2 = 4 & x = \pm 2 \end{cases}$$

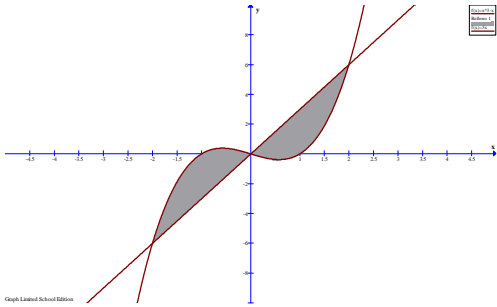
$$A = 2 \int_1^2 ((x^2 - 4) - (x^4 - 4x^2)) dx = 2 \left[\frac{-x^5}{5} + \frac{5x^3}{3} - 4x \right]_1^2 =$$

$$2 \left[\left(\frac{-32}{5} + \frac{40}{3} - 8 \right) - \left(\frac{-1}{5} + \frac{5}{3} - 4 \right) \right] = 2 \left[\frac{-31}{5} + \frac{35}{3} - 4 \right] = \frac{44}{15} u.a.$$

27

Trobeu l'àrea que tanquen les corbes: $y = x^3 - x$, $y = 3x$.

RAONAMENT



Punts de tall entre les dues funcions

$$\begin{cases} y = x^3 - x \\ y = 3x \end{cases} \Rightarrow x^3 - x = 3x \Rightarrow \begin{cases} x = 0 \\ x^2 = 4 \end{cases} \Rightarrow \begin{cases} x = 0 \\ x = \pm 2 \end{cases}$$

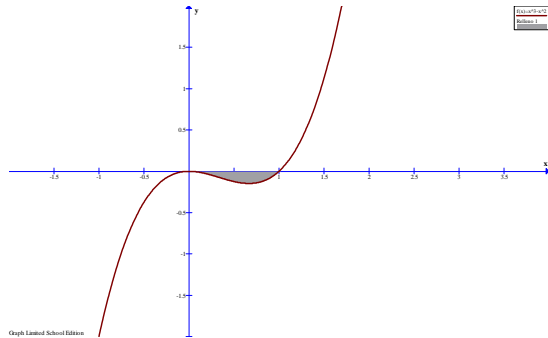
$$A = 2 \int_0^2 [(3x) - (x^3 - x)] dx = 2 \left[\frac{-x^4}{4} + 2x^2 \right]_0^2 = 2 \left[(-4 + 8) - \left(\frac{-1}{4} + 2 \right) \right]$$

$$= \frac{9}{2} \text{ u.a.}$$

28

Àrea compresa entre $y = x^3 - x^2$ i l'eix OX.

RAONAMENT



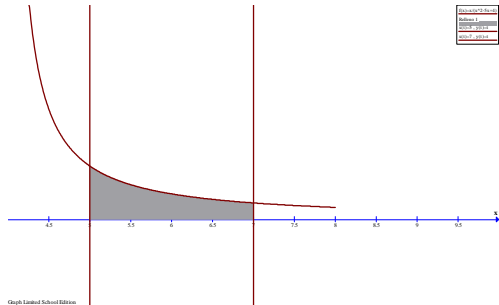
Punts de tall amb l'eix d'abscisses $\begin{cases} y = x^3 - x^2 \\ y = 0 \end{cases} \Rightarrow \begin{cases} x^2 = 0 \\ x = 1 \end{cases} \Rightarrow \begin{cases} x = 0 \\ x = 1 \end{cases}$

$$A = \int_0^1 -(x^3 - x^2) dx = \left[-\frac{x^4}{4} + \frac{x^3}{3} \right]_0^1 = \left(-\frac{1}{4} + \frac{1}{3} \right) - 0 = \frac{1}{12} \text{ u.a.}$$

29

Trobeu l'àrea compresa entre la funció $y = \frac{x}{x^2 - 5x + 4}$ i les rectes: $x = 5$ i $x = 7$.

RAONAMENT



$$\frac{x}{(x-1)(x-4)} = \frac{A}{x-1} + \frac{B}{x-4} \Rightarrow A(x-4) + B(x-1) = x \Rightarrow$$

$$\begin{cases} x=4 & 0+3B=4 \\ x=1 & -3A+0=1 \end{cases} \Rightarrow \begin{cases} B = \frac{4}{3} \\ A = -\frac{1}{3} \end{cases}$$

$$F(x) = \int \frac{x}{(x-1)(x-4)} dx = A \ln|x-1| + B \ln|x-4| =$$

$$= -\frac{1}{3} \ln|x-1| + \frac{4}{3} \ln|x-4|$$

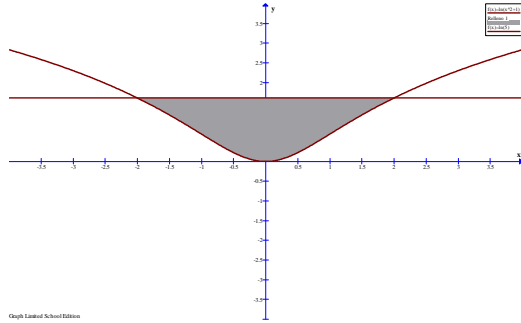
$$A = \int_5^7 \frac{x}{x^2 - 5x + 4} dx = F(7) - F(5) = -\frac{1}{3} \ln 6 + \frac{4}{3} \ln 3 + \frac{1}{3} \ln 4 - 0 \text{ u.a.}$$

30

Àrea que tanquen les corbes: $y = \ln(x^2 + 1)$ i $y = \ln 5$

* $\operatorname{arctg}(-\alpha) = -\operatorname{arctg}(\alpha)$.

RAONAMENT



Punts de tall entre les funcions $\begin{cases} y = \ln(x^2 + 1) \\ y = \ln 5 \end{cases} \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

$$A = 2 \int_0^2 (\ln 5 - \ln(x^2 + 1)) dx = [2x \ln 5]_0^2 - 2 \int_0^2 \ln(x^2 + 1) dx$$

$$4 \ln 5 - 4 \ln 5 + 8 - 4 \operatorname{arctg} 2 = \boxed{8 - 4 \operatorname{arctg} 2} \text{ u.a.}$$

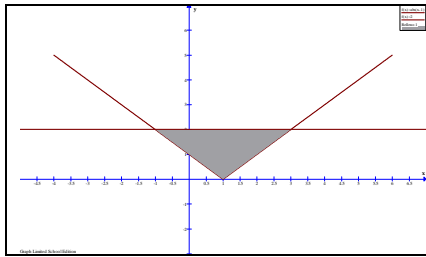
$$\begin{cases} F(x) = \int \ln(x^2 + 1) dx = x \ln(x^2 + 1) - 2 \int \frac{x^2}{x^2 + 1} dx = \\ * \begin{cases} = x \ln(x^2 + 1) - 2x + 2 \operatorname{arctg}(x) + c \\ \left\{ \begin{array}{l} u = \ln(x^2 + 1) \quad du = \frac{2x}{x^2 + 1} dx \\ dv = dx \quad v = x \end{array} \right. \end{cases} \end{cases}$$

$$F(2) - F(0) = (2 \ln 5 - 4 + 2 \operatorname{arctg} 2) - 0 = 2 \ln 5 - 4 + 2 \operatorname{arctg} 2$$

EXERCICIS PROPOSATS

31

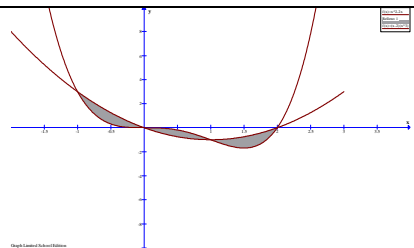
Àrea que tanquen les corbes $y = |x - 1|$ i $y = 2$.



Sol.
 $A = 4 \text{ u.a.}$

32

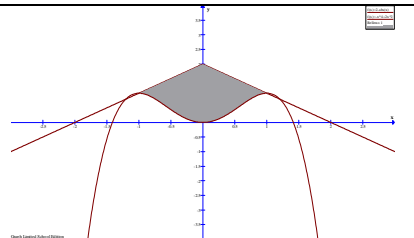
Àrea que tanquen les corbes: $y = x^2 - 2x$ i $y = x^3(x - 2)$



Sol.
 $A = \frac{4}{15} \text{ u.a.}$

33

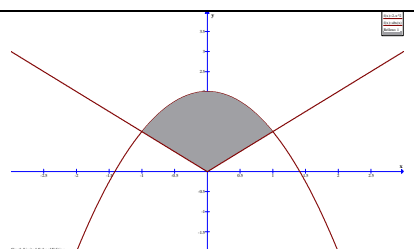
Àrea que tanquen les corbes: $y = -x^4 + 2x^2$, $y = x + 2$, $y = -x + 2$.



Sol.
 $A = \frac{31}{15} \text{ u.a.}$

34

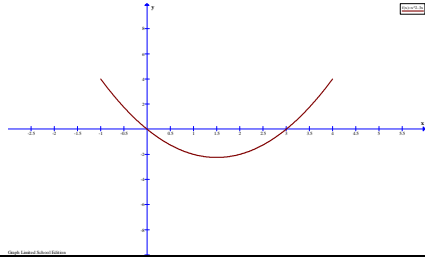
Àrea que tanquen les funcions: $y = 2 - x^2$ i $y = |x|$.



Sol.
 $A = \frac{7}{3} \text{ u.a.}$

35

Calculeu l'àrea del recinte limitat per la corba $y = x^2 - 3x$ i l'eix OX

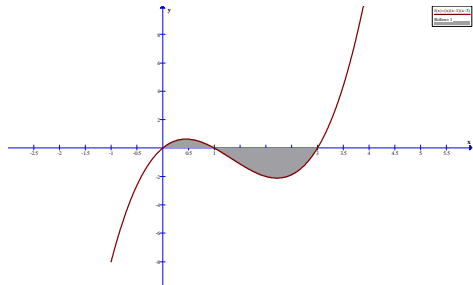


Sol.

$$A = \frac{9}{2} u.a.$$

36

Calculeu l'àrea del recinte limitat per la corba $y = x(x-1)(x-3)$ i l'eix OX .

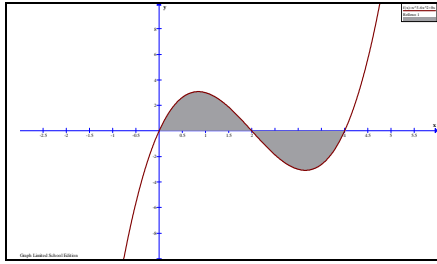


Sol.

$$A = \frac{37}{12} u.a.$$

37

Calculeu l'àrea del recinte limitat per la corba $y = x^3 - 6x^2 + 8x$ i l'eix OX .



Sol.
 $A = 8u.a.$